Modeling of Short-Pulse Laser Radiation in Terms of Photon Wave Function in Coordinate Representation

A. Davydov [1], T. Zlydneva [2]

[1] Dept. of Applied and Theoretical Physics of Physical-Mathematical Faculty,
Nosov Magnitogorsk State Technical University, Magnitogorsk, Russia
E-mail: ap-dav@yandex.ru
[2] Dept. of Applied Mathematics and Informatics of Physical-Mathematical Faculty,
Nosov Magnitogorsk State Technical University, Magnitogorsk, Russia
E-mail: tapazl@yandex.ru

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Abstract. On the base of the photon's wave function (wave packet) in Schrödinger coordinate representation developed in previous papers the modeling of the propagation in space and in time of a wave packet of the photon corresponding to a separate laser impulse of radiation of femtosecond diapason is carried out. The development is proceeded from single-particle approach with the usage of the basis eigenfunctions of energy, momentum and helicity operators in continuous momentum spectrum case. The wave packet is constructed in the form of superposition of these eigenfunctions as the integral over the all momentum space. In the case of Gaussian momentum distribution the character of an extension of a form of this wave packet is established. The essence of wave-particle duality of light and microparticles is specified. The hypothesis is formulated, however, that the photon is a quasi-particle corresponding to the propagation of spin wave in physical vacuum. It is claimed that this question is related to the structure of the leptons and other particles on the Planck distances.

Keywords: Schrödinger equation; Maxwell's equations; wave packet; detector; extreme maximon

INTRODUCTION

Since publication of work [1] it is considered (see also [2–8]) that the wave function of a photon cannot be constructed in configuration space although in momentum representation it is applied in many aspects. The reason of it consists in the zero mass of "rest" of a photon. Nevertheless, for a photon it is also possible to construct wave function ("a wave packet") if it is intended for the indication of density of probability not of localization of a photon, for example in the spirit of electron in atom, but of its detection in space. In modern experiments (transfer of optical signals on quantum communication channels, "quantum teleportation", "paradoxes" with single photons, etc.) there is a need for association of photons with the localized carriers of elementary units of information. Therefore creation of wave function of a photon in coordinate representation becomes again topical "at the new level of knowledge". Then, knowing the wave function, it is also possible with the quantum-mechanical point of view to come to an explanation of interference, diffraction and polarization of electromagnetic waves. Without doing the full review here, we will refer on [2–14] where anyway this subject is touched, the term "wave function of a photon" is used, but nevertheless wave function of a photon, normalized on unit probability, isn't given in coordinate representation. Apparently, the first works in which the idea of a possible "localization" of a photon described by a probability density, defined by the normalized per unit of the wave function, have been proposed are [15–18]. Further development of the theory and justification of building of coordinate single-particle wave function of the photon was performed in [19–25] and others.

The purpose of this article is the modeling of the propagation in space and in time of a wave packet of the photon corresponding to a separate laser impulse of radiation of femtosecond diapason. This wave packet is the photon wave function in the coordinate representation.

PHOTON WAVE FUNCTION IN COORDINATE REPRESENTATION

We give here some details of the construction and interpretation of the photon wave function in coordinate representation accordantly to [22, 25].

The construction of the photon wave function in coordinate representation is based on the synthesis of classical electrodynamics and quantum mechanics taking into account the correspondence principle. Maxwell's equations in the Majorana form [26], for the vectors $\boldsymbol{\xi}=\mathbf{E}+i\mathbf{H}$ and $\boldsymbol{\eta}=\mathbf{E}-i\mathbf{H}$, made up of the electric $(\mathbf{E})$ and magnetic $(\mathbf{H})$ fields intensities (in Gaussian System), are taken as the initial:

$$i\hbar\frac{\partial\xi}{\partial t}=c(\hat{\mathbf{s}}\hat{\mathbf{p}})\xi,\quad i\hbar\frac{\partial\eta}{\partial t}=-c(\hat{\mathbf{s}}\hat{\mathbf{p}})\eta,\quad (\hat{\mathbf{p}}\boldsymbol{\xi})=0,\quad (\hat{\mathbf{p}}\boldsymbol{\eta})=0, \tag{1}$$

where $\hat{\mathbf{p}}=-i\hbar\hat{\nabla}$ is the operator of the momentum of the particle; c is the velocity of light in vacuum; $\hat{\mathbf{s}}$ is the operator of the photon spin in vector representation:

$$\begin{aligned}\hat{\mathbf{s}}&=\mathbf{e}_x\hat{s}_x+\mathbf{e}_y\hat{s}_y+\mathbf{e}_z\hat{s}_z=\\&=\mathbf{e}_x\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}+\mathbf{e}_y\begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix}+\mathbf{e}_z\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}=i\begin{pmatrix}0&-\mathbf{e}_z&\mathbf{e}_y\\\mathbf{e}_z&0&-\mathbf{e}_x\\-\mathbf{e}_y&\mathbf{e}_x&0\end{pmatrix}.\end{aligned} \tag{2}$$

The vectors $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ in a matrix form have an appearance:

$$\xi=\begin{pmatrix}\mathrm{E}_x+i\mathrm{H}_x\\\mathrm{E}_y+i\mathrm{H}_y\\\mathrm{E}_z+i\mathrm{H}_z\end{pmatrix},\quad \eta=\begin{pmatrix}\mathrm{E}_x-i\mathrm{H}_x\\\mathrm{E}_y-i\mathrm{H}_y\\\mathrm{E}_z-i\mathrm{H}_z\end{pmatrix}, \tag{3}$$

and should be considered as independent [7]. For a bivector $\Phi_{\mathrm{bv}}=\begin{pmatrix}\xi\\\eta\end{pmatrix}$ it is necessary to solve the equation which is the generalization of Dirac equation for a massless particle with spin $s=1$ in the "standard" or "bivector" representations. In the latter case it has the form:

$$i\hbar\frac{\partial\Phi_{\mathrm{bv}}}{\partial t}=\hat{H}_{\mathrm{bv}}\Phi_{\mathrm{bv}}\quad\text{or}\quad i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\xi\\\eta\end{pmatrix}=\frac{c}{s}\begin{pmatrix}(\hat{\mathbf{s}}\hat{\mathbf{p}})&0\\0&-(\hat{\mathbf{s}}\hat{\mathbf{p}})\end{pmatrix}\begin{pmatrix}\xi\\\eta\end{pmatrix}. \tag{4}$$

Thus together with the solution of the equation (4) *the task is formulated to find the eigenfunctions* Φ_{bv} *and eigenvalues of the mutually commuting operators of the complete set:*

$$\left\{\hat{E}=i\hbar\,\partial/\partial t;\ \hat{H}_{\mathrm{bv}}=c(\hat{\boldsymbol{\alpha}}_{\mathrm{bv}}\hat{\mathbf{p}});\ \hat{\mathbf{p}}=-i\hbar\hat{\nabla};\ \hat{\Lambda}\right\}, \tag{5}$$

where the matrix $\hat{\boldsymbol{\alpha}}_{\text{bv}}$, operators of the helicity $\hat{\Lambda}$ and of the spin $\hat{\mathbf{S}}$ of a photon in bivector representation equal:

$$\hat{\boldsymbol{\alpha}}_{\text{bv}} = \begin{pmatrix} \hat{\mathbf{s}} & 0 \\ 0 & -\hat{\mathbf{s}} \end{pmatrix}, \quad \hat{\Lambda} = \frac{(\hat{\mathbf{S}}\hat{\mathbf{p}})}{sp} = \frac{(\hat{\mathbf{S}}\hat{\mathbf{p}})}{p} = \frac{1}{p}\begin{pmatrix} (\hat{\mathbf{s}}\hat{\mathbf{p}}) & 0 \\ 0 & (\hat{\mathbf{s}}\hat{\mathbf{p}}) \end{pmatrix},, \quad \hat{\mathbf{S}} = \begin{pmatrix} \hat{\mathbf{s}} & 0 \\ 0 & \hat{\mathbf{s}} \end{pmatrix}. \tag{6}$$

The solution of this task consists in the following [22, 25]:

1) To states of a photon with *positive* energy $E^{(+)}(k) = \hbar kc = +pc$ (which consistent with the special theory of relativity [27]) the orthonormal bivectors, answering to a helicity $\lambda = \pm 1$, are:

$$\Phi^{(+)}_{\text{bv};\mathbf{k},+1}(\mathbf{r},t) = \begin{pmatrix} \xi^{(+)}_{\mathbf{k},+1}(\mathbf{r},t) \\ 0 \end{pmatrix} = \frac{(\text{Oe})\,\mathrm{e}_{+1}(\mathbf{k})}{(2\pi)^{3/2}} e^{i(\mathbf{kr}-kct)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{7}$$

$$\Phi^{(+)}_{\text{bv};\mathbf{k},-1}(\mathbf{r},t) = \begin{pmatrix} 0 \\ \eta^{(+)}_{\mathbf{k},-1}(\mathbf{r},t) \end{pmatrix} = \frac{(\text{Oe})\,\mathrm{e}_{-1}(\mathbf{k})}{(2\pi)^{3/2}} e^{i(\mathbf{kr}-kct)} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{8}$$

respectively, where (Oe) is unit of measure (Oersted) of values $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$.

2) To states of a photon with negative energy $E^{(-)}(k) = -\hbar kc = -pc$ (which theoretically possible) the orthonormal bivectors, answering to the helicity $\lambda = \mp 1$, are:

$$\Phi^{(-)}_{\text{bv};\mathbf{k},-1}(\mathbf{r},t) = \begin{pmatrix} \xi^{(-)}_{\mathbf{k},-1}(\mathbf{r},t) \\ 0 \end{pmatrix} = \frac{(\text{Oe})\,\mathrm{e}_{-1}(\mathbf{k})}{(2\pi)^{3/2}} e^{i(\mathbf{kr}+kct)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{9}$$

$$\Phi^{(-)}_{\text{bv};\mathbf{k},+1}(\mathbf{r},t) = \begin{pmatrix} 0 \\ \eta^{(-)}_{\mathbf{k},+1}(\mathbf{r},t) \end{pmatrix} = \frac{(\text{Oe})\,\mathrm{e}_{+1}(\mathbf{k})}{(2\pi)^{3/2}} e^{i(\mathbf{kr}+kct)} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{10}$$

respectively, where the complex polarization vectors $\mathbf{e}_\lambda(\mathbf{k}) = \left[\mathbf{e}_I(\mathbf{k}) + i\lambda\,\mathbf{e}_{II}(\mathbf{k})\right]/\sqrt{2}$, and $\mathbf{e}_I$, $\mathbf{e}_{II}$ are the real mutually orthogonal unit vectors forming a right-handed triad with a vector $\mathbf{n} = \mathbf{k}/k$ for a given $\mathbf{k} = \mathbf{p}/\hbar$:

$$|\,\mathbf{e}_I| = |\,\mathbf{e}_{II}| = 1, \quad (\mathbf{e}_I\,\mathbf{n}) = (\mathbf{e}_{II}\,\mathbf{n}) = (\mathbf{e}_I\,\mathbf{e}_{II}) = 0, \quad \mathbf{e}_{II} = \left[\mathbf{n}\times\mathbf{e}_I\right], \tag{11}$$

$$\mathbf{n} = \left[\mathbf{e}_I \times \mathbf{e}_{II}\right] = i\lambda\left[\mathbf{e}_\lambda \times \mathbf{e}^*_\lambda\right] = \lambda \mathrm{e}^+_\lambda(\mathbf{k})\,\hat{\mathbf{s}}\,\mathrm{e}_\lambda(\mathbf{k}), \tag{12}$$

that gives the orthonormality for $\mathbf{e}_\lambda$ and useful relations *if* vector $\mathbf{e}_I$ *does not its change at change of sign of vector* $\mathbf{n}$, namely:

$$(\mathbf{e}^*_{\lambda'}\,\mathbf{e}_\lambda) = \delta_{\lambda'\lambda}, \quad \mathrm{e}^+_{\lambda'}\mathrm{e}_\lambda = \delta_{\lambda'\lambda}, \quad \mathbf{e}_\lambda(\mathbf{n}) = \mathbf{e}_{-\lambda}(-\mathbf{n}), \quad [\,\mathbf{e}_\lambda(\mathbf{k})]^* = \mathbf{e}_{-\lambda}(\mathbf{k}) = \mathbf{e}_\lambda(-\mathbf{k}). \tag{13}$$

Owing to (13) the orthonormality relations take place for bivectors (7)–(10):

$$\int d^3\mathbf{r}\left[\Phi^{(\pm)}_{\text{bv};\,\mathbf{k}',\lambda'}(\mathbf{r},t)\right]^+ \Phi^{(\pm)}_{\text{bv};\,\mathbf{k},\lambda}(\mathbf{r},t) = (\text{Oe})^2\,\delta_{\lambda'\lambda}\delta(\mathbf{k}'-\mathbf{k}), \tag{14}$$

$$\int d^3\mathbf{r}\left[\Phi^{(\pm)}_{\text{bv};\,\mathbf{k}',\lambda'}(\mathbf{r},t)\right]^+ \Phi^{(\mp)}_{\text{bv};\,\mathbf{k},\lambda}(\mathbf{r},t) = 0. \tag{15}$$

Then it is postulated that the photon states with the positive and negative energy, and given by $\mathbf{E}$ *and* $\mathbf{H}$ *are the superposition of bivectors of monochromatic plane waves* (7)–(10):

$$\Phi_{\text{bv}}^{(\pm)}(\mathbf{r},t) \equiv \Phi_{\text{bv};\,\pm1}^{(\pm)}(\mathbf{r},t) + \Phi_{\text{bv};\,\mp1}^{(\pm)}(\mathbf{r},t) \equiv$$

$$\equiv \int B(\mathbf{k},\pm1)\,\Phi_{\text{bv};\mathbf{k},\,\pm1}^{(\pm)}(\mathbf{r},t)\,d^3\mathbf{k} + \int \left[B(-\mathbf{k},\mp1)\right]^*\,\Phi_{\text{bv};\,\mathbf{k},\,\mp1}^{(\pm)}(\mathbf{r},t)\,d^3\mathbf{k}. \tag{16}$$

Using specified bivectors, satisfying the necessary conditions of orthonormality and completeness, it is possible to write the spatial density distribution of energy of the photon in the state (16) and the wave function $\Psi^{(\pm)}(\mathbf{r},t)$ of this state, normalized per unit probability:

$$\rho_E^{(\pm)}(\mathbf{r},t) \equiv \frac{1}{8\pi}\left[\Phi_{\text{bv}}^{(\pm)}(\mathbf{r},t)\right]^+ \Phi_{\text{bv}}^{(\pm)}(\mathbf{r},t) = \frac{1}{8\pi}\sum_{\lambda}\left[\Phi_{\text{bv};\,\lambda}^{(\pm)}(\mathbf{r},t)\right]^+ \Phi_{\text{bv};\,\lambda}^{(\pm)}(\mathbf{r},t) =$$

$$= \frac{1}{8\pi}\left\{\left[\xi_{\pm1}^{(\pm)}(\mathbf{r},t)\right]^+ \xi_{\pm1}^{(\pm)}(\mathbf{r},t) + \left[\eta_{\mp1}^{(\pm)}(\mathbf{r},t)\right]^+ \eta_{\mp1}^{(\pm)}(\mathbf{r},t)\right\} \equiv \frac{1}{8\pi}\left\{\left|\boldsymbol{\xi}_{\pm1}^{(\pm)}(\mathbf{r},t)\right|^2 + \left|\boldsymbol{\eta}_{\mp1}^{(\pm)}(\mathbf{r},t)\right|^2\right\} \equiv$$

$$= \frac{1}{8\pi}\left\{\left[\mathbf{E}_{\xi,\pm1}^{(\pm)}(\mathbf{r},t)\right]^2 + \left[\mathbf{H}_{\xi,\pm1}^{(\pm)}(\mathbf{r},t)\right]^2 + \left[\mathbf{E}_{\eta,\mp1}^{(\pm)}(\mathbf{r},t)\right]^2 + \left[\mathbf{H}_{\eta,\mp1}^{(\pm)}(\mathbf{r},t)\right]^2\right\}, \tag{17}$$

$$\Psi^{(\pm)}(\mathbf{r},t) = \int b(\mathbf{k},\pm1)\,\Psi_{\mathbf{k},\,\pm1}^{(\pm)}(\mathbf{r},t)\,d^3\mathbf{k} + \int \left[b(-\mathbf{k},\pm1)\right]^*\,\Psi_{\mathbf{k},\mp1}^{(\pm)}(\mathbf{r},t)\,d^3\mathbf{k}, \tag{18}$$

where

$$b(\mathbf{k},\lambda) = \frac{(\text{Oe})}{\sqrt{8\pi\hbar kc}}B(\mathbf{k},\lambda), \qquad \Psi_{\mathbf{k},\lambda}^{(\pm)}(\mathbf{r},t) = \frac{1}{(\text{Oe})}\Phi_{\text{bv};\mathbf{k},\lambda}^{(\pm)}(\mathbf{r},t). \tag{19}$$

Thus the $\Psi^{(\pm)}(\mathbf{r},t)$ and dimensionless functions $\Psi_{\mathbf{k},\lambda}^{(\pm)}(\mathbf{r},t)$ satisfy the normalization

$$\int d^3\mathbf{r}\left[\Psi_{\mathbf{k},\lambda}^{(\pm)}(\mathbf{r},t)\right]^+ \Psi_{\mathbf{k}',\lambda'}^{(\pm)}(\mathbf{r},t) = \delta_{\lambda'\lambda}\delta(\mathbf{k}'-\mathbf{k}), \tag{20}$$

$$\int d^3\mathbf{r}\left[\Psi^{(\pm)}(\mathbf{r},t)\right]^+ \Psi^{(\pm)}(\mathbf{r},t) = \int d^3\mathbf{r}\,\rho_P^{(\pm)}(\mathbf{r},t) = 1. \tag{21}$$

The photon wave function $\Psi^{(\pm)}(\mathbf{r},t)$ in the state of the wave packet (18) with both positive and negative energy, satisfies the Schrodinger equation of the form (4), from which the continuity equation follows for density of probability $\rho_P^{(\pm)}(\mathbf{r},t)$ and of stream density $\mathbf{j}_P^{(\pm)}(\mathbf{r},t)$ of probability to find the photon in the vicinity of the point $\mathbf{r}$ in a timepoint t:

$$\frac{\partial\rho_P^{(\pm)}(\mathbf{r},t)}{\partial t} + \operatorname{div}\mathbf{j}_P^{(\pm)}(\mathbf{r},t) = 0, \tag{22}$$

where

$$\rho_P^{(\pm)}(\mathbf{r},t) = \left[\Psi^{(\pm)}(\mathbf{r},t)\right]^+ \Psi^{(\pm)}(\mathbf{r},t), \quad \mathbf{j}_P^{(\pm)}(\mathbf{r},t) = c\left[\Psi^{(\pm)}(\mathbf{r},t)\right]^+ \hat{\boldsymbol{\alpha}}_{\text{bv}}\Psi^{(\pm)}(\mathbf{r},t). \tag{23}$$

The wave function in the momentum representation corresponds to the wave function (18), namely:

$$\Psi^{(\pm)}(\mathbf{k},t) \equiv \left\langle \mathbf{k} \middle| \Psi^{(\pm)} \right\rangle \equiv \frac{1}{(2\pi)^{3/2}} \int e^{-i\mathbf{kr}}\, \Psi^{(\pm)}(\mathbf{r},t)\, d^3\mathbf{r} =$$

$$= e^{\mp ikct} \left\{ b(\mathbf{k},\pm 1)\, e_{\pm 1}(\mathbf{k}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left[b(-\mathbf{k},\mp 1) \right]^* e_{\mp 1}(\mathbf{k}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}. \quad (24)$$

If coefficients $b(\mathbf{k},\lambda)$ are known, then using wave functions (18), (24) it is possible to calculate all characteristics of a free photon. For example, average value of energy of a photon is defined as:

$$\overline{E^{(\pm)}} \equiv \left\langle \Psi^{(\pm)} \middle| \hat{E}\, \Psi^{(\pm)} \right\rangle = \left\langle \sum_{\lambda'} \Psi_{\lambda'}^{(\pm)} \middle| \hat{E} \sum_{\lambda} \Psi_{\lambda}^{(\pm)} \right\rangle =$$

$$= \int \left(\pm \hbar kc \right) \left\{ \left| b(\mathbf{k},\pm 1) \right|^2 + \left| b(-\mathbf{k},\mp 1) \right|^2 \right\} d^3\mathbf{k} = \int E^{(\pm)}(k)\, \rho_P^{(\pm)}(\mathbf{k})\, d^3\mathbf{k}. \quad (25)$$

This formula gives the value coinciding with the one defined in classical electrodynamics, and also the formulas (17). This fact expresses the correspondence principle causing the introduction of the wave function of the photon in a condition of a wave packet (18).

EVOLUTION IN SPACE AND TIME OF PHOTON WAVE PACKET CORRESPONDING TO A SINGLE FEMTOSECOND LASER PULSE

On the basis of the above stated general method of construction of wave function of a free photon in coordinate representation in [28] the most important wave packet in scientific and methodical aspects with Gaussian momentum distribution is considered.

In order to more fully reveal the physical content and functionality of the wave packet (18) we will choose the coefficients of this packet having the simplest, "Gaussian" form:

$$b(\mathbf{k},\pm 1) = \left[b(-\mathbf{k},\mp 1) \right]^* = \sqrt{\frac{\alpha_1\alpha_2\alpha_3}{2\pi\sqrt{\pi}}}\, \exp\left[-\frac{1}{2}\left(\alpha_1^2 k_x^2 + \alpha_2^2 k_y^2 + \alpha_3^2 (k_z \mp k_0)^2 \right) - i\mathbf{kr}_0 \right], \quad (26)$$

where parameters $\mathbf{k}_0 = (0,0,k_0)$, $\mathbf{r}_0 = (x_0, y_0, z_0)$, α_1, α_2, α_3 characterize average values and dispersions of the corresponding physical quantities in the state of a photon (18) and satisfy the normalization condition (21).

Parameterization (26) answers to the state of a photon with a zero average helicity as the values $\lambda = \pm 1$ are presented in (26) with equal probability. All characteristics of this wave packet can be broken into two categories: 1) momentum-energy, expressed only through the parameters appearing in (26), and 2) space-time, for calculation of which it is required to set still polarization vectors $\mathbf{e}_\lambda(\mathbf{k})$. According to statements of quantum mechanics, values of these two categories of characteristics answer to the corresponding uncertainty relations. New here, compared to the quantum mechanics of particles with mass, is the fact that the values of characteristics of the second category essentially can depend on "choice" of vectors $\mathbf{e}_\lambda(\mathbf{k})$.

Momentum-energy characteristics

Applying (18), (26) and quantum mechanical formula of calculation of average value of physical quantity F operator of which is equal $\hat{F}$:

$$\overline{F^{(\pm)}} \equiv \left\langle \Psi^{(\pm)} \middle| \hat{F}\, \Psi^{(\pm)} \right\rangle = \left\langle \sum_{\lambda'} \Psi_{\lambda'}^{(\pm)} \middle| \hat{F} \sum_{\lambda} \Psi_{\lambda}^{(\pm)} \right\rangle, \tag{27}$$

where $\Psi_{\lambda}^{(\pm)}$ at the values $\lambda = \pm 1$ are defined by corresponding terms of the formula (18), we find at once average values of projections of momentum and their squares:

$$\overline{p_x^{(\pm)}} = \overline{p_y^{(\pm)}} = 0, \quad \overline{p_z^{(\pm)}} = \pm \hbar k_0, \tag{28}$$

$$\overline{\left(p_x^{(\pm)}\right)^2} = \frac{\hbar^2}{2\alpha_1^2}, \quad \overline{\left(p_y^{(\pm)}\right)^2} = \frac{\hbar^2}{2\alpha_2^2}, \quad \overline{\left(p_z^{(\pm)}\right)^2} = \frac{\hbar^2}{2\alpha_3^2} + \hbar^2 k_0^2, \tag{29}$$

from where it follows that the average vector of a momentum of the photon in a state of positive energy is directed along the axis *z*, and with negative is opposite to it:

$$\overline{\mathbf{p}^{(\pm)}} = \pm \hbar \mathbf{k}_0 \equiv \pm \hbar k_0 \mathbf{e}_z . \tag{30}$$

Dispersions of projections of a momentum on the axes are defined by $\alpha_1, \alpha_2, \alpha_3$:

$$D_{p_x} \equiv \overline{\left(p_x^{(\pm)}\right)^2} - \left(\overline{p_x^{(\pm)}}\right)^2 = \frac{\hbar^2}{2\alpha_1^2}, \quad D_{p_y} = \frac{\hbar^2}{2\alpha_2^2}, \quad D_{p_z} = \frac{\hbar^2}{2\alpha_3^2}. \tag{31}$$

Then uncertainty of photon momentum projections an in state (18) are reduced to formulas:

$$\Delta p_x \equiv \sqrt{D_{p_x}} = \frac{\hbar}{\alpha_1 \sqrt{2}}, \quad \Delta p_y = \frac{\hbar}{\alpha_2 \sqrt{2}}, \quad \Delta p_z = \frac{\hbar}{\alpha_3 \sqrt{2}}, \tag{32}$$

where the presence $\sqrt{2}$ is connected with such choice which gives, according to (24) the simplest form of the momentum distribution in the state (18), namely Gaussian form:

$$\rho_P^{(\pm)}(\mathbf{k}) = \left| b(\mathbf{k}, \pm 1) \right|^2 + \left| b(-\mathbf{k}, \mp 1) \right|^2 = \frac{\alpha_1 \alpha_2 \alpha_3}{\pi \sqrt{\pi}} \exp\left[-\alpha_1^2 k_x^2 - \alpha_2^2 k_y^2 - \alpha_3^2 (k_z \mp k_0)^2 \right] \tag{33}$$

Below we present the appropriate formulas in the case when $\alpha_1 = \alpha_2 = \alpha_3$.

Applying the formula (25), (33), we find the average energy of the photon in state (18), respectively with positive and negative spectrum of its energy:

$$\overline{E^{(\pm)}} = \pm \hbar k_0 c \left[\left(1 + \frac{1}{2\alpha_1^2 k_0^2} \right) \operatorname{erf}(\alpha_1 k_0) + \frac{\exp(-\alpha_1^2 k_0^2)}{\alpha_1 k_0 \sqrt{\pi}} \right], \tag{34}$$

and also, similarly, the mean square of energy of the photon in the state (18):

$$\overline{\left(E^{(\pm)}\right)^2} = c^2 \hbar^2 k_0^2 \left(1 + \frac{3}{2\alpha_1^2 k_0^2} \right). \tag{35}$$

Using (34) and (35), it is possible to calculate dispersion and uncertainty of energy of a photon in state (18) with momentum distribution (33), according to the general definition:

$$D_E = \overline{\left(E^{(\pm)}\right)^2} - \left(\overline{E^{(\pm)}}\right)^2, \quad \Delta E = \sqrt{D_E}. \tag{36}$$

Space-time characteristics

Requirements (11)–(13) are satisfied, e.g., for the following vectors [22, 25]:

$$e_I(\mathbf{k}) = \begin{pmatrix} 1-(1-\cos\theta)\cos^2\varphi \\ -(1-\cos\theta)\sin\varphi\cos\varphi \\ -\sin\theta\cos\varphi \end{pmatrix}, \quad e_{II}(\mathbf{k}) = \begin{pmatrix} -(1-\cos\theta)\sin\varphi\cos\varphi \\ \cos\theta+(1-\cos\theta)\cos^2\varphi \\ -\sin\theta\sin\varphi \end{pmatrix}, \text{ at } 0 \le \theta \le \frac{\pi}{2}, \tag{37}$$

$$e_I(\mathbf{k}) = \begin{pmatrix} 1-(1+\cos\theta)\cos^2\varphi \\ -(1+\cos\theta)\sin\varphi\cos\varphi \\ \sin\theta\cos\varphi \end{pmatrix}, \quad e_{II}(\mathbf{k}) = \begin{pmatrix} (1+\cos\theta)\sin\varphi\cos\varphi \\ \cos\theta-(1+\cos\theta)\cos^2\varphi \\ -\sin\theta\sin\varphi \end{pmatrix}, \text{ at } \frac{\pi}{2} < \theta \le \pi, \tag{38}$$

where the Cartesian components of the corresponding vectors in the usual configuration space are specified, expressed in terms of the spherical coordinates of vector **k** in momentum space. Taking into account formulas (37), (38) it is also conveniently to carry out the calculation of space-time characteristics in momentum representation, using the formula (24).

In particular for average values of coordinates and their squares of a point of detection of the photon in state (18) we obtain the following expressions:

$$\overline{x^{(\pm)}} = x_0, \quad \overline{y^{(\pm)}} = y_0, \quad \overline{z^{(\pm)}} = z_0 \pm ct\,\overline{n_z^{(\pm)}}, \tag{39}$$

$$\overline{\left(x^{(\pm)}\right)^2} = x_0^2 + \frac{\alpha_1^2}{2} + A_1^{(2)} + c^2t^2\,\overline{\left(n_x^{(\pm)}\right)^2}, \quad \overline{\left(y^{(\pm)}\right)^2} = y_0^2 + \frac{\alpha_1^2}{2} + A_2^{(2)} + c^2t^2\,\overline{\left(n_y^{(\pm)}\right)^2}, \tag{40}$$

$$\overline{\left(z^{(\pm)}\right)^2} = z_0^2 + \frac{\alpha_1^2}{2} + A_3^{(2)} + c^2t^2\,\overline{\left(n_z^{(\pm)}\right)^2} \pm 2ct\,\overline{n_z^{(\pm)}}\,z_0, \tag{41}$$

where:

$$\overline{n_z^{(\pm)}} = \left[\left(1 - \frac{1}{2\alpha_1^2k_0^2}\right)\operatorname{erf}(\alpha_1k_0) - \frac{\exp(-\alpha_1^2k_0^2)}{\alpha_1k_0\sqrt{\pi}}\right], \tag{42}$$

$$\overline{\left(n_x^{(\pm)}\right)^2} = \overline{\left(n_y^{(\pm)}\right)^2} = \frac{1}{2}\left[1 - \overline{\left(n_z^{(\pm)}\right)^2}\right] = \frac{1}{2\alpha_1^2k_0^2}\left[1 - \frac{\sqrt{\pi}}{2\alpha_1k_0}\operatorname{erfi}(\alpha_1k_0)\exp(-\alpha_1^2k_0^2)\right], \tag{43}$$

$$A_1^{(2)} = A_2^{(2)} = -\frac{1}{2}A_3^{(2)} + \Delta A_{13}^{(2)}, \quad \Delta A_{13}^{(2)} \equiv 2\alpha_1^2\exp\left(-\alpha_1^2k_0^2\right)\int_0^1 \frac{\exp\left(\alpha_1^2k_0^2u^2\right)}{1+u}\,du, \tag{44}$$

$$A_3^{(2)} = \frac{1}{2k_0^2}\left[\frac{1+2\alpha_1^2k_0^2}{2\alpha_1k_0}\operatorname{erfi}(\alpha_1k_0)\exp\left(-\alpha_1^2k_0^2\right)\sqrt{\pi} - 1\right]. \tag{45}$$

From (39)–(45) it follows that the dispersions D_x, D_y, D_z of coordinates of a point of detection of the photon which is in the state (18) parameterized by means of (26) are equal:

$$D_x = D_y = \frac{\alpha_1^2}{2} + A_1^{(2)} + c^2 t^2 \overline{\left(n_x^{(\pm)}\right)^2} = \frac{\alpha_1^2}{2} + A_1^{(2)} + c^2 t^2 D_{n_x}, \tag{46}$$

$$D_z = \frac{\alpha_3^2}{2} + A_3^{(2)} + c^2 t^2 \left\{ \overline{\left(n_z^{(\pm)}\right)^2} - \left(\overline{n_z^{(\pm)}} \right)^2 \right\} = \frac{\alpha_3^2}{2} + A_3^{(2)} + c^2 t^2 D_{n_z}. \tag{47}$$

According to (46) the dispersions D_x and D_y are the same for the considered wave packet which is symmetric relatively of the z axis.

Analysis of modeling results

As seen from (46), the rate of expansion of the wave packet is the same in each plane xy, in accordance with the fact that the wave packet (18) with the parameterization (26) remains symmetric relatively of the z axis. As characteristics of speed of this expansion it is possible to use periods τ_x, τ_y, τ_z during which initial dispersions (at $t = 0$) are doubled along the directions x, y, z. From (46)–(47) we find:

$$\tau_x = \frac{\Delta x(t=0)}{c\,\Delta n_x} = \tau_y = \frac{\Delta y(t=0)}{c\,\Delta n_y}, \qquad \tau_z = \frac{\Delta z(t=0)}{c\,\Delta n_z}. \tag{48}$$

Since even in a simple form of distribution (26) it is not possible, analytically to obtain an expression for the probability density in configuration space, we carry out the analysis of the evolution of the considered wave packet by means of calculation of the intensity of electric field, using the initial formula (3), (18), (19). Not equal to zero in this case is only a projection of intensity E_x, which characterizes a certain way, the spatial probability density. At $\alpha_1 = \alpha_1 = \alpha_3$ the spatial "form" of a wave packet in the initial time is "spherical". We will give results of numerical calculation for the packet corresponding to the duration of 80 fs radiation with the central wavelength of 10 microns. On an axis of symmetry of a packet, the density of probability of photon detection in the vicinity of the center of the packet moves practically with velocity of light in vacuum. The farther from the axis, the lower the velocity of probability density is in the direction of the average velocity (along) of wave packet.

Thus, there is a transformation of the original shape of the wave packet in a certain "conical" shape (see. Fig. 1 and 2).

Speed of this transformation is the more, the less initial "radius" of a wave packet (18), according to the general representations of quantum mechanics. Fig. 1 and 2 show the distributions of the most significant projection of the intensity of electric field E_x computed respectively in two different moments of time: $t = \tau_z$ and $t = 2\tau_z$, where τ_z is the time of expansion of the packet (48) along an z axis.

THE MAIN FORMULA OF WAVE-PARTICLE DUALITY AND NATURE OF PHOTON

In our opinion the constructed photon quantum mechanics substantially removes a problem of wave-particle duality of quantum "particles". Main "formula" of wave-particle duality of light and microparticles can be formulated as follows [29]:

1. Photons and microparticles at interaction behave as a corpuscle, transferring and transmitting (to other particles) in a certain quantity as dynamic characteristics (energy, momentum, angular momentum), and "internal" (mass, electric charge, spin, etc.). In particular, such transfer is carried out at hit of a photon or microparticle in quite dot detector

(or a point on the screen) with coordinate $\mathbf{r}$ at time point t. The fact of hit of "all particle entirely" in the dot detector is characteristic for a corpuscle, but not for some real wave.

2. However, photons and microparticles propagate in space by "wave rules", that is their distribution in space is described by wave function. In particular, density of probability of detection in space of the nonrelativistic particle with a nonzero mass is postulated by a formula $\rho(\mathbf{r},t)=|\Psi(\mathbf{r},t)|^2$, and a photon by (23). This probability density also causes the hit of a photon and microparticle in the dot detector. A characteristic interferential picture on the screen corresponds to distribution of $\rho(\mathbf{r},t)$ along the screen.

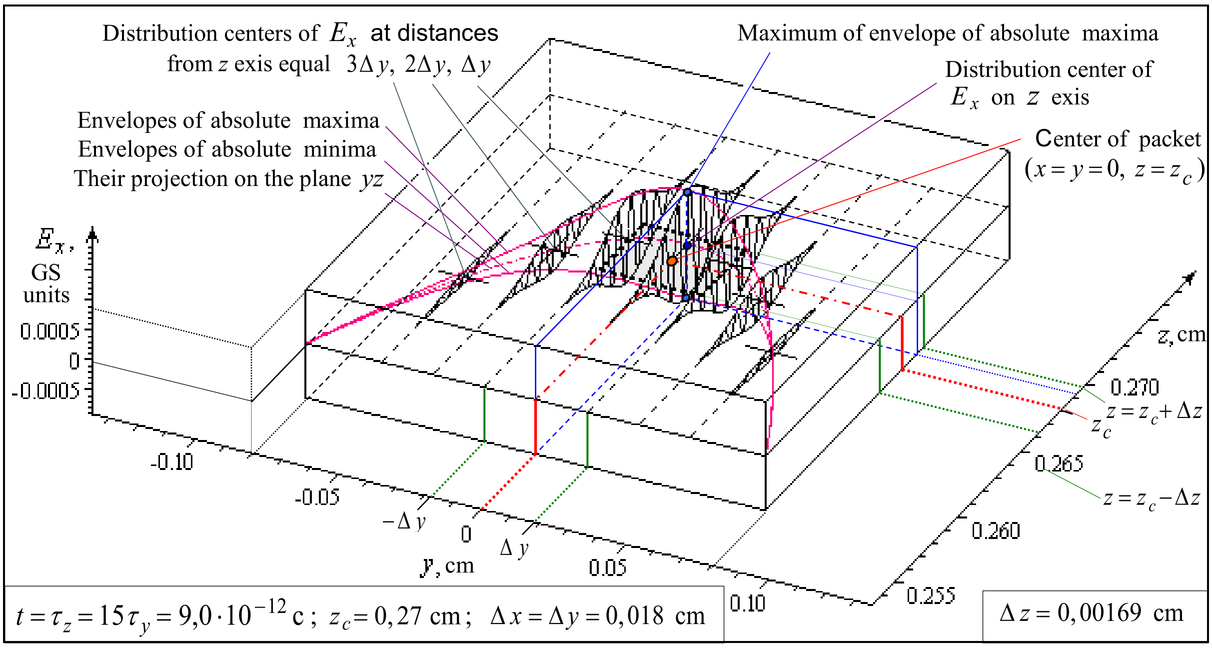

Figure 1. Distribution of electric field intensity E_x at time moment $t = \tau_z$

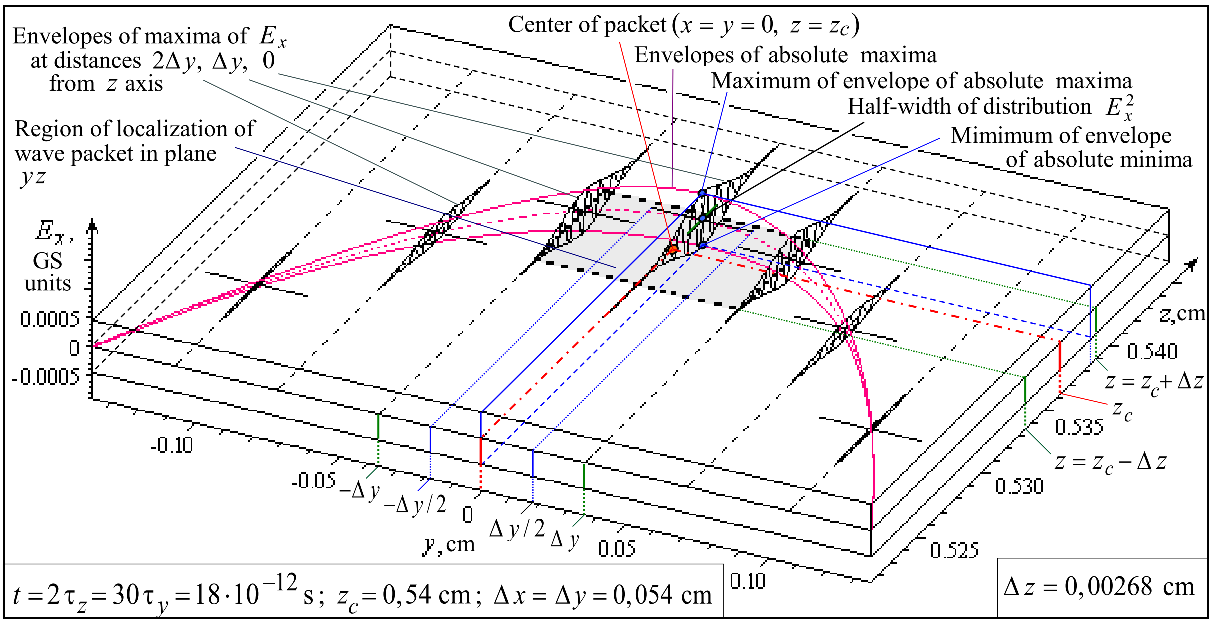

Figure 2. Distribution of electric field intensity E_x at time moment $t = 2\tau_z$

Nevertheless, electromagnetic radiation even in the case of waves small lengths and obviously expressed "corpuscular properties", is *impossible* to consider as a stream of the certain "created", "dot" particles, similar to the massive particles. In our view, the photon is a quasi-particle, and light is a result of the propagation of a spin wave in physical vacuum, the structure and nature of which have to be considered at the Planck distance [25, 30]. This question is closely related to the structure of the leptons and other fundamental particles on the same distances. According to [31–33] the center of an electron is extreme maximon, that is the quantum nonsingular object creating round itself an extreme Kerr-Newman metric. It has spin $s=1/2$ and approximately Planck mass, charge and radius. For most observed phenomena involving photons it is possible to give the following interpretation of their propagation in vacuum. In the photon propagation the middle-ordered (in time and space) alternate spins flip occurs of virtual vacuum extreme maximons, which creates the effect of the spin wave, and in "macroscopic scale" produce manifestation the corpuscular-wave properties of photons. However due to the vector dominance also exhibited by the photons, their propagation in vacuum can be associated also with other, more complex virtual processes.

CONCLUSION

The results of our modeling of photon wave packet propagation allow to illustrate the possibility of a single-photon approach to the description of electromagnetic phenomena. In particular, it appears that those aspects of interference and diffraction such as the interference pattern of Young's double-slit experiment, which were described in the language of classical electrodynamics, obviously can be described in the language of quantum mechanics without the involvement of the apparatus of second quantization of the electromagnetic field. This significantly expands the scope of “ordinary” quantum mechanics and considerably reduces the problem of wave-particle duality in the present level of our knowledge.

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