Non-Linear Programming in the Synthesis of Regulators

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Abstract. The article deals with the synthesis of regulators. There are presented the opportunity of using the non-linear programming in the synthesis the control systems by real interpolation method.

Keywords: regulator, control object, transfer function, real interpolation method, non-linear programming

INTRODUCTION

The modern control objects have very complex structure. In this connection regulators also increases complexity. At the same time, it is known that satisfactory quality of regulation can be obtained using relatively simple controls often. In addition, the simple controls are more preferable then complex. Since they are easier to understand and configuration, and it is less demanding of computing resources. Although some of these problems of complex regulators is gradually reduced in transition to digital implementation, the problem of creating regulators of reduced order is still relevant, and attracts the attention of researchers.

Many approaches to the construction of regulators can be divided into two groups. In the first case, the regulator was projected with a simplified structure. Initially its order is below the order of the control object. For example, there are all methods of PID-regulators synthesis. Sometimes, this process is known as setting regulator.

In the second case, full regulator is calculated, then its order is reduced to a predetermined or minimum possible without significant loss of basic properties of a closed-loop system.

For the second approach detailed model and a good formalization of the control object need. Thus, the system is configured in such a way has more control accuracy. And it is possible to arbitrarily set the desired requirements for the synthesized system. In the first approach there is no such possibility in principle.

Thus, the problem of the synthesis of the regulator minimum order is current in the theory of control for complex systems.

The paper presents a method for the synthesis regulators, including minimum order, based on the real interpolation method and nonlinear programming.

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The real interpolation method for synthesis the control systems

The real interpolation method (RIM) [1, 2] refers to a group of operator methods. From the classical approach it different views of direct integral transformation. The method uses a real transformation, that is the transition from the original function $f(t)$ to the image-function $F(\delta)$, that has real variable δ.

The formula for the real image $F(\delta)$ it follows directly from the formula of the Laplace transform. There the complex variable s replaced by real variables δ:

$$F(\delta) = \int_0^\infty f(t) \cdot e^{-\delta t} dt, \ \ \delta \in [0, \infty). \tag{1}$$

Thus the function $f(t)$ has the natural limitations. It must be continuous, equal to zero for all values of t in the interval $(-\infty;0]$. And it must be integral absolutely:

$$\int_0^\infty |f(t)|^2 dt < \infty.$$

The image-function of the $f(t)$ can be obtained in an analytical form of a simple replacement the relevant formula Laplace complex variable s to a real variable δ or as a graph $F(\delta)$ and a set of samples $F(\delta_i)$, which are called numerical characteristics. Thus numerical characteristic takes full information about the original model. With regard to the synthesis of the regulator using the model in the form of numerical characteristics makes it easy to realize the computing aspects of this procedure.

Solution of the problem synthesis of the regulator be the RIM is based on an approximate equality of the numerical characteristics of the synthesized the regulator $W_{reg}(\delta)$ and dividing the numerical characteristics of the desired model $W_{des}(\delta)$ to the numerical characteristics of the control object $W_{OU}(\delta)$:

$$\{W_{reg}(\delta_i)\} = \{W_{des}(\delta_i) / W_{OU}(\delta_i)\}, \ \ i = \overline{1, n}, \tag{2}$$

where n – the number of points of the numerical characteristics, which are called interpolation nodes [1].

The key role in the process of synthesis played of the approximate numerical characteristics. The quality of this procedure can influence choice of interpolation nodes and the principle of approximation. And also the introduction of restrictions on the coefficients $W_{reg}(s)$ impact on it.

Normally, the number of interpolation nodes n is chosen so that it was possible to form a square a system of algebraic equations and solve it relatively the unknown control parameters [1]. It is useful to use a much larger number of points, focusing not on the exact solution of algebraic equations, and approximate solution.

Absence of restrictions on the coefficients $W_{reg}(s)$ is result in unstable or non-robust closed-loop system. It is known that the maximum robustness of the system is achieved by using the minimum-phase regulators. It is important for unstable objects especially.

All noted problems can be solved using the procedure of nonlinear programming in the Matlab.

Statement and solution synthesis problem based the RIM as a nonlinear programming problem

Nonlinear programming problem in the program FMINCON of the Optimization Toolbox Matlab section posed as a problem of finding the minimum of the nonlinear problem:

$$\min_x f(x)$$

with restrictions:

$$c(x) < 0;$$

$$ceq(x) = 0;$$

on condition that:

$$Ax \le b,$$

$$Aeg \cdot x = beg,$$

$$lb \le x \le ub,$$

where x – vector of unknowns; b – vector of put limits on inequality, beg – vector point constraints of equality, lb, ub – vectors of restrictions above and below, A – matrix inequality constraints; Aeg – constraint matrix equations, $c(x)$ and ceq(x) – the function of non-linear constraints.

Let the numerical characteristic of the dividing the numerical characteristics of the desired model to the control object is given in the form n samples p_i in interpolation nodes δ_i. The regulator is calculated as:

$$W_{reg}(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + 1}. \quad (3)$$

The problem of synthesis the regulator can be formulated as a nonlinear programming problem. It is required to minimize the functional $f(x) = e^T \cdot He$ under conditions:

$$b_2\delta_1^2 + b_1\delta_1 + b_0 - p_1\delta_1^2 a_2 - p_1\delta_1 a_1 + e_1 = p_1,$$

$$\dots$$

$$b_2\delta_n^2 + b_1\delta_n + b_0 - p_n\delta_n^2 a_2 - p_n\delta_n a_1 + e_n = p_n,$$

$$b_2 \ge 0,$$

$$b_1 \ge 0,$$

$$b_0 \ge 0,$$

$$a_2 \ge 0,$$

$$a_1 \ge 0.$$

Vector e is the discrepancy of the approximate solution of a system of linear equations in the interpolation nodes. The vector of unknown x include the regulator parameters b_2, b_1, b_0, a_2, a_1 and the components of the vector of discrepancies e.

The restrictions are imposed only on parameters b_2, b_1, b_0, a_2, a_1.

Apply the proposed approach for the control system with object of type:

$$W_{OU}(s) = \frac{15}{s^2 + s + 15}.$$

In [4] the synthesis of the compensation regulator of the second order was considered. It provides in a closed system, the specified quality and accuracy of static. Design a regulator minimal order using nonlinear programming.

The result of solving the problem non-linear programming using the program FMINCON obtained regulator in type:

$$W_{reg}(s) = \frac{0.02s + 0.265}{0.06s + 1}.$$

Graph of transient processes is shown in Fig. 2. It shows that the obtained regulator has provided parameters of quality, close to the required quality.

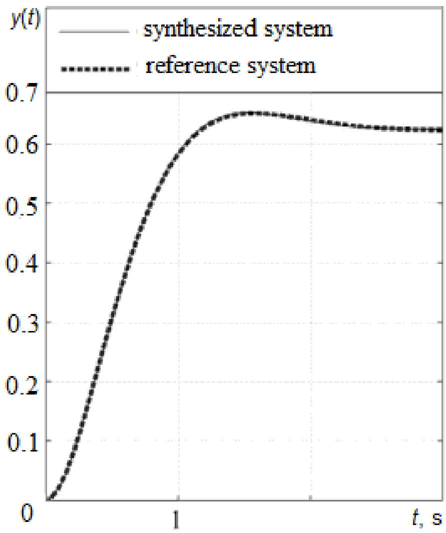

Figure 2. The transient processes in the synthesized and the reference systems for minimal-phase control object

The analysis shows that the special benefit from approximations and restrictions on signs of the coefficients of synthesized regulator in the case of the minimum-phase objects of the second order do not have. However, the situation changes drastically during the transition to non-minimal-phase objects.

Consider the non-minimal-phase object of the second order of the form:

$$W_{OU}(s) = \frac{-s + 15}{s^2 + s + 15}.$$

It has one zero in the right-half of the s-plane.

The structure of the regulator and view the desired transfer function remained the same.

The result of solving the problem non-linear programming using the program FMINCON obtained regulator in type:

$$W_{reg}(s) = \frac{0.0148s + 0.4270}{0.001s + 1}.$$

Graph of transient processes is shown in Fig. 3.

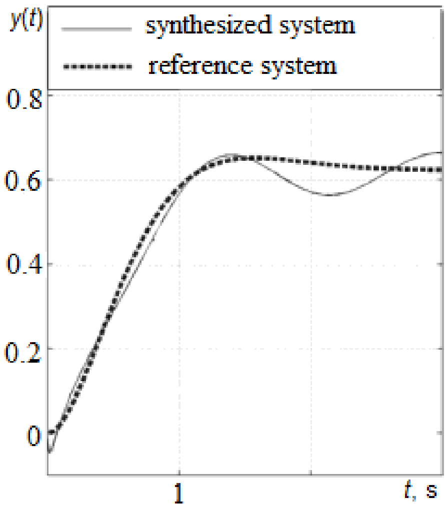

Figure 3. The transient processes in the synthesized and the reference systems for non-minimal-phase control object

Figure 3 shows that the obtained regulator provided stability in a closed-loop system with non-minimal-phase control object without compensation, but the quality of the transition process is not fully compliant. The transition process is a great time setting.

In the case of unconstrained optimization for synthesis regulator system is not stable because right pole of control object is compensated right zero of regulator.

Conclusion

In the present paper we investigated the possibility of using real interpolation method combined with nonlinear programming tools for solving the problem of synthesis of regulator. There is a possibility of the synthesis regulator, which provides the stability of the closed-loop control system for minimum-phase, and for non-minimal-phase control object.

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