Monte Carlo Simulation Model for Estimation of Reliability Indexes of Electronic Means

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Abstract. The Monte Carlo method is a well-known technique for solving a verity of stochastic problems. Development of a simulation model based on this method provides flexibility for analyzing investigated processes and estimated parameters. In this paper, the Monte Carlo method was used for construction of a simulation model for the operational process of electronic medium for the purpose of integrated reliability index estimation. Multi-state functioning process considers a set of exploitation factors, such as sudden, gradual, latent and fictitious failures, human factor of service staff and time parameters of preventive maintenance. Implementing simulation experiments with the proposed model allows analyzing the reliability of electronic through popular indexes, coefficient of operation efficiency and availability coefficient, taking into account important factors of the operational process.

Keywords: simulation, Monte Carlo, Semi Markov process, reliability indexes

INTRODUCTION

Modern electronic devices consist of multiple components which are susceptible to aging and deterioration and that could lead to sudden and gradual failures of the whole system or its parts. These circumstances are a reason for system's reliability decreasing. One of the well-known method for reliability rising is preventive maintenance (PM), which main objective is to keep allowable reliability level through operations implemented periodically or deepening on the current system's condition. In both cases maintenance are performed by service staff, which could have different level of professional skills that could exert influence on reliability. Besides, preventive operations are implemented during a time that is not generally a constant; also a variety of diagnostic device could be used for testing and checking of fault state that cause appearance of latent and fictitious failures. All mentioned factors define multiple quantity of states of operation process for analyzed system and as a result it can affect reliability characteristics such as coefficient of operation efficiency (K_{OE}) and availability coefficient (K_A) [1]. Reliability analysis for basic operation process model with two main states, operable and non-operable, is enough well-known and could be easily performed with analytical solution [1]. Realization of analytical model with consideration of abovementioned factors could be implemented using theory of Markov and Semi Markov process [2, 3]. But such approach is not always a convenient analysis way because it is necessary to make routine operations for input of some changes into the model. Simulation technique seems to be more attractive method for studying of observable system's process and to obtain estimations of reliability characteristics taking into account factors which are inherent in real situations. In this paper, a simulation model based on Embedded Markov chain of Semi Markov process in combination with a Monte Carlo technique is proposed to calculate integrated reliability indexes: coefficient of operation efficiency and availability coefficient. The model gives an opportunity to obtain easily estimations of the reliability indexes depending on the abovementioned factors and to use it for further optimization of PM parameters.

METHODOLOGY

Development of a simulation model is a well-known process and usually consists of the sequence of stages, such as conceptual description of the researched process, its mathematical formalization, development of simulation algorithm and its implementation [4, 5]. These steps are presented in the sections below.

Conceptual Description

Conceptual description of researched process is one of the steps for development of any mathematical model and includes relevant information about its peculiarities [4]. In this paper we use important knowledge in order to provide necessary comprehension.

Functioning process of electronic consists of two stages: normal operation and active ageing. The service staff implements PM at the point of normal operation in order to keep workable condition and required reliability level. PM is performed with periodicity T_{int} and intended to prevent partially sudden failures, but generally gradual failures. This kind of failures is preceded by a misalignment of system's parts in irreversible physicochemical processes of components, in other words due to ageing. Gradual failures of components could amount 30–80 percentages of total quantity of failures. Consequently, accounting of misalignment state of repairable systems is necessary to analyze of its reliability. Time parameters of PM are defined according to branch documentation and usually are presented by the following quantities: testing time (t_t), time for system tuning and configuration (t_a), fault search time (t_s) and emergency repair time (t_r). Systems' parameters control and diagnosis could be implemented by different measurement devices and automated diagnosis apparatuses. Validity of diagnostic devices information could be estimated by probability of erroneous determination of real system state.

The following assumptions were made for development of the conceptual model:

1. The electronic equipment is subjected to misalignments during its exploitation and misalignments' rate and failures of detuned electronics subordinate to exponential law.
2. Misalignment of equipment isn't detected during its exploitation.
3. Failures of diagnostic devices aren't taken into account.
4. Recovery time and duration of maintenance are constants.
5. Repair starts immediately after a failure occurs or it is detected by diagnostic devices.
6. Service staff can make errors during PM which could result to failures.
7. Probability of service staff errors vanishes while the maintenance intervals tend to zero.

Electronic medium could be in the following states during its operational process:

1. Operable state (*S1*).
2. Misalignment state (*S2*).
3. Non-operable state (*S3*).
4. PM of operative system (*S4*).
5. Maintenance of system with misalignment (*S5*).
6. Latent failure (*S6*).

7. Maintenance of system being in latent failure (*S7*).
8. Fictitious failure (*S8*).

Sojourn time of *S1* state has stochastic character and is determined by distribution function and failure rate and misalignment rate of system. System passes to *S2* state if one of its parameters exceeds its assumed value. Non-operable state *S3,* due to an obvious failure, defines system's inability to perform required function in consequence of a failure of its component. The system stay in this state during necessary time to be repaired, which includes testing time t_t, fault search time t_s and emergency repair time t_r.

The system passes to *S4*, *S5* and *S7* states at the schedule date according the maintenance system for testing and adjusting of equipment if the last is necessary. Testing is an operation for confirmation of operable system's state or detection of misalignment or identification of latent failure state and it's characterized by duration of testing time t_t. Adjusting is a set of operations for recovery of normative values of system's parameters. It could be realized replacing components leading to misalignment, as well as tuning parameters during time t_a using, for example, adjusting elements or automated tuning facilities. Testing and tuning are performed only in *S5* state. Maintenance operations are carried out by operating staff, therefore, during testing t_t and adjusting time t_a an error could be made leading to equipment's failure and its transition to *S3* state. Distribution function of probability of operating staff errors $F_p(T_{int})$ is offered to use in order to take into account this factor.

Monitoring electronic medium conditions is usually performed by external and embedded diagnostic devices (DD). Besides, built-in DD implement periodic diagnostic system' modules and fix failures using beeps and/or fault light. External DD are exploited during implementing of PM procedures. The main DD's parameters are I type (α) and II type (β) errors.

Diagnosis error α is a probability to declare the operable device as "out of order". Error β is a probability to declare the inoperable device as "faultless". Diagnostic devices which are used for maintenance have higher grade of accuracy than embedded medium, therefore, $\alpha_1 > \alpha_2$, $\beta_1 > \beta_2$. For the stochastic process under study, the diagnosis errors are supposed not to be equal to zero, consequently, there are states of latent failure and fictitious failure. In *S6* state the system stands until performing of maintenance procedures, and during it operative staff can detect the malfunction. In *S8* state the system passes on when embedded DD makes an error giving a signal about fictions failure. Besides, it's assumed that instantaneous testing is executed during time t_t, after the system proceeded to operable state *S1*.

Thus, if it is possible to obtain statistical data for each state of the process then the coefficient of operation efficiency K_{OE} could be calculated as:

$$K_{OE} = \frac{T_{OS}}{T_{OS} + T_{RS} + T_{MS}}, \qquad (1)$$

where T_{OS}, T_{RS} and T_{MS} – mean time of operable, repair and maintenance states, accordingly.

The expression for availability coefficient K_A can be written as:

$$K_A = \frac{T_{OS}}{T_{OS} + T_{RS}}. \qquad (2)$$

Generally, next step of the mathematical model development is interpretation of conceptual description with an illustrative graphical model. Graphical interpretation is presented in Fig. 1 by way of a graph with 8 states and transitions between states with accordance to the values of one-step transition probabilities P_{ij}, $i \in [1, 8]$, $j \in [1, 8]$.

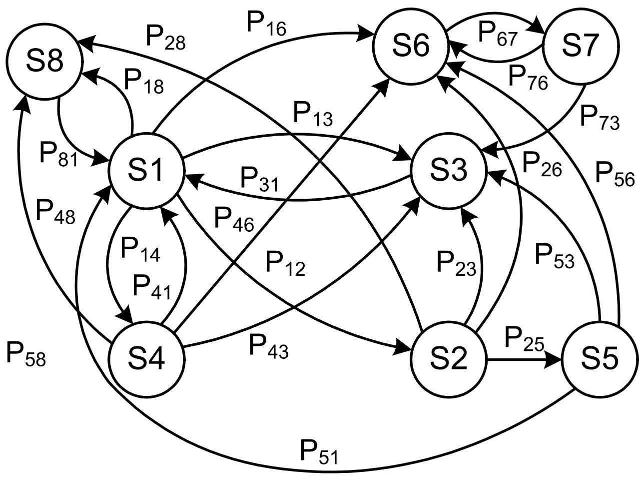

Figure 1. Transition state graph of the functioning process of repairable and maintainable electronic medium

Mathematical Formulation

According to conceptual description and graphical model, the observable operational could be presented as a random sequence of transitions from current state S_i to next state S_j. An example of such transitions in time domain is shown in Fig. 2.

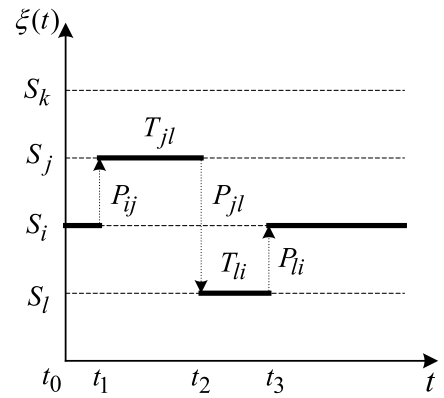

Figure 2. Time dependence of stochastic transitions

Theory of Markov and Semi Markov is used for mathematical formalization of transition sequence $\xi(t)$ [2, 3]. According to the theory this sequence of stochastic transitions could be described by Markov chain with continuous time and discreet state space. Furthermore, Markov property is obligatory to be hold true. This property, inherent in exponential law, however, sojourn time complies with this law only in state *S1* and *S2*. In the rest of states the duration of stay is a constant. Therefore, was suggested to use the model of the model of Embedded Markov chain of Semi Markov process [2, 3]. Application of simulation technique gives opportunity to take into account different character of input parameters without difficult routine operations with analytical formulas' manipulations [4].

The model's set of parameters for the observable process is presented as:

- vector of initial states:

$$P_0 = \left\{ P_1^0, P_2^0, P_3^0, P_4^0, P_5^0, P_6^0, P_7^0, P_8^0 \right\}; \tag{3}$$

- square matrix of transition probabilities:

$$P=\begin{vmatrix} 0 & \begin{matrix}(1-F_{13})\times\\ \times F_{12}\end{matrix} & \begin{matrix}(1-\beta_1)\times\\ \times F_{13}\end{matrix} & \begin{matrix}(1-\alpha_1)\times\\ \times(1-F_{13})\\ \times(1-F_{12})\end{matrix} & 0 & \begin{matrix}\beta_1\times\\ \times F_{13}\end{matrix} & 0 & \begin{matrix}\alpha_1\times\\ \times(1-F_{13})\times\\ \times(1-F_{12})\end{matrix} \\ 0 & 0 & \begin{matrix}(1-\beta_1)\times\\ \times F_{23}\end{matrix} & 0 & \begin{matrix}(1-\alpha_1)\times\\ \times(1-F_{23})\end{matrix} & \begin{matrix}\beta_1\times\\ \times F_{23}\end{matrix} & 0 & \alpha_1(1-F_{23}) \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \begin{matrix}(1-\alpha_2)\times\\ \times(1-F_p)\end{matrix} & 0 & \begin{matrix}(1-\beta_2)\times\\ \times F_p\end{matrix} & 0 & 0 & \begin{matrix}\beta_2\times\\ \times F_p\end{matrix} & 0 & \alpha_2(1-F_p) \\ \begin{matrix}(1-\alpha_2)\times\\ \times(1-F_p)\end{matrix} & 0 & \begin{matrix}(1-\beta_2)\times\\ \times F_p\end{matrix} & 0 & 0 & \begin{matrix}\beta_2\times\\ \times F_p\end{matrix} & 0 & \alpha_2(1-F_p) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1-\beta_2 & 0 & 0 & \beta_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}, \qquad (4)$$

where F_{12} – exponential distribution function of transition probability from *S1* to *S2* with periodicity equals T_{int}; F_{13} – exponential distribution function of transition probability from *S1* to *S3* with periodicity equals T_{int}; F_{23} – exponential distribution function of transition probability from *S2* to *S3* with periodicity equals T_{int}; $F_p(T_{int})$ – distribution function of service staff errors probability.

- vector of density functions:

$$f=\left\{(\lambda_{13}+\lambda_{12})e^{-(\lambda_{13}+\lambda_{12})T_1},\lambda_{23}e^{-\lambda_{23}T_2},t_t+t_s+t_r,t_t,t_t+t_a,T_{int},t_t,t_t\right\}, \qquad (5)$$

where λ_{12}, λ_{13} – rates of misalignments and sudden failures, respectively, hours^{-1}; λ_{23} – rate of sudden failures of misaligned system, hours^{-1}; t_t – testing time, hours; t_a – time for system configuration, hours; t_s – fault search time, hours; t_r – emergency repair time, hours.

Thus, multiple simulations of transition sequences from S_i state to S_j in accordance with abovementioned model provide opportunity to collect statistical data about system’s operational process and to estimate values of integrated reliability indexes K_{OE} and K_A.

Simulation Algorithm

The simulation algorithm was developed applying discreet-event approach [5]. According to this method, the process is implemented only in its significant states, and transitions between them are realized by means of specially organized statistical sampling procedure which is based on Monte Carlo method [3, 6]. The technique includes three main steps:

1. Determining the first state of the process described by Semi Markov model according to the initial states’ vector (3).
2. Calculating the system’s duration of stay in current state using the vector (5) before to pass to another state. In order to perform this operation with specified density function could be used the inverse transforms method [5];
3. Defining the next state S_j according to the square matrix of transition probabilities (4) when the following inequality is correct:

$$\sum_{j=1}^{f-1}P_{ij}<u\le\sum_{j=1}^{f}P_{ij},\; i=const,\; j=\overline{1,8}, \qquad (5)$$

where u – uniformly distributed number within the range of 0–1.

After correcting the expression (5), target value of *j*-index equals to variable *f*. By way of the random values generator was used L'Ecuyer algorithm with period of reiteration of pseudorandom numbers about 10^{18} [7]. The flow chart of the algorithm, taking into account the periodicity of preventive procedures as the main parameter of maintenance system are presented in Fig. 3

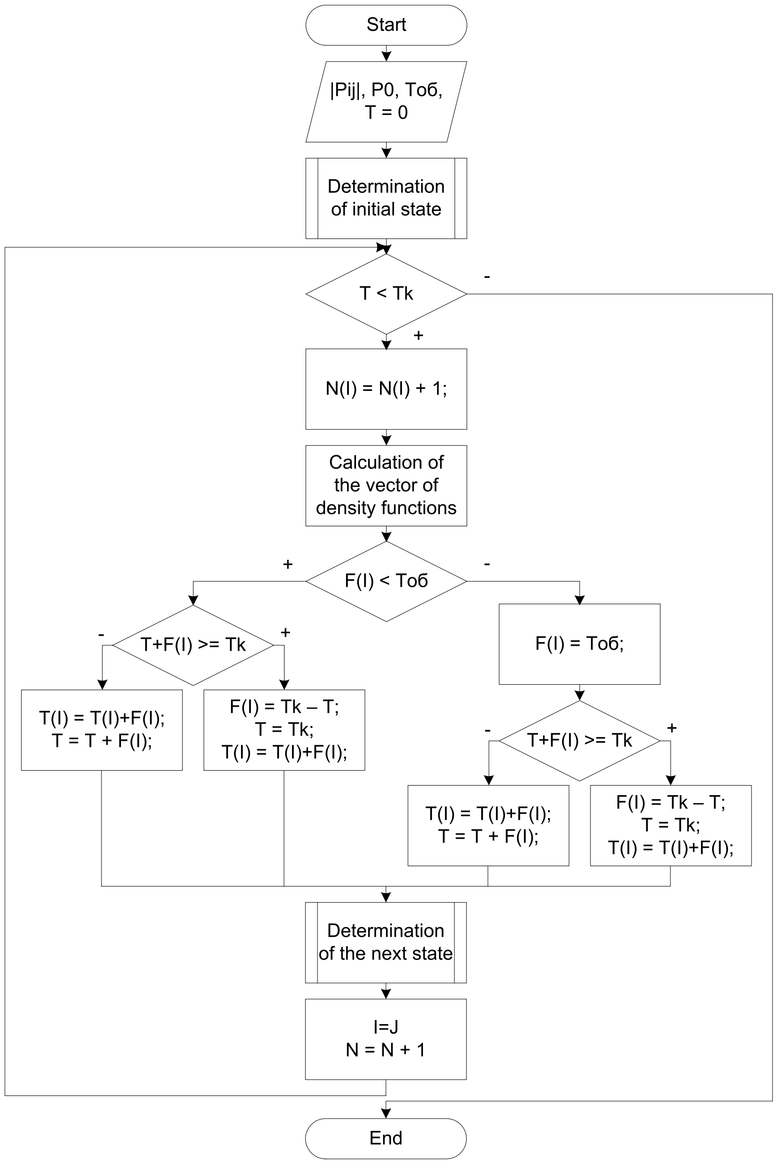

Figure 3. Flow chart of the simulation algorithm

Thus, if we can simulate the investigated process using the described algorithm during time T_k for each value of periodicity of maintenance $T_{int} \in [0, T_k]$ with step ΔT then it is possible to calculate sojourn time for each process's state S_i, $i \in [1, 8]$. On basis of collected statistical data, the estimations of integrated reliability indexes $\overline{K_{OE}}(T_{int})$ and $\overline{K_A}(T_{int})$ are implemented with specified accuracy ε and confidence probability Q:

$$\overline{K_{OE}^{m}}\left(T_{int}^{m}\right)=\frac{1}{N_m}\sum_{j=1}^{N_m}\frac{T_1^j\left(T_{int}^m\right)+T_2^j\left(T_{int}^m\right)+T_8^j\left(T_{int}^m\right)}{\sum\limits_{i=1}^{8}T_i^j\left(T_{int}^m\right)},\ m=\overline{1,M}; \tag{6}$$

$$\overline{K_A^m}\left(T_{int}^m\right)=\frac{1}{N_m}\sum_{j=1}^{N_m}\frac{T_1^j\left(T_{int}^m\right)+T_2^j\left(T_{int}^m\right)+T_8^j\left(T_{int}^m\right)}{\sum_{i=1}^{3}T_i^j\left(T_{int}^m\right)+T_6^j\left(T_{int}^m\right)+T_8^j\left(T_{int}^m\right)},\ m=\overline{1,M},\qquad(7)$$

where T^j_i – total stay time in S_i for j replication of the model; M – quantity of points for calculation determined as $T_k/\Delta T$; N_m – quantity of replications of simulation model for each estimated point of the dependences $\overline{K_{OE}}\left(T_{int}\right)$ and $\overline{K_A}\left(T_{int}\right)$ as the calculation is performed using different amount of sampling of simulation experiment which is organized by automatic stop principle with achievement of specified accuracy ε [8].

The simulation model, on the basis of the suggested algorithm, was developed in C++ programming language. As a language selection criterion, the effectiveness and usability were considered, and also the possibility to interact with Matlab software was taken into account. Thus, the model was prepared as a mex-function that provides opportunity to perform some functions, for example, statistical data processing, using the library of the Matlab system. However, the model's part demanding a lot of machine time due to applying the Monte Carlo method is realized on C++ language, that significantly reduce the time for implementation of simulation experiments.

SIMULATION EXPERIMENT

As an example the simulation experiment was performed with the following specified parameters: $\lambda_{12} = 50\cdot10^{-6}$ hours^{-1}, $\lambda_{13} = 5\cdot10^{-6}$ hours^{-1}, $\lambda_{23} = 0.5\lambda_{12}$, $\alpha_1 = 0.01$, $\alpha_2 = 0.005$, $\beta_1 = 0.01$, $\beta_2 = 0.005$, $t_t = 2$ hours, $t_a = 1$ hours, $t_s = 3$ hours, $t_r = 3$ hours, $F_p(T_{int}) = 0$, $T_k = 10^6$ hours and ΔT =600 hours. The results of the experiment are illustrated in Fig. 4.

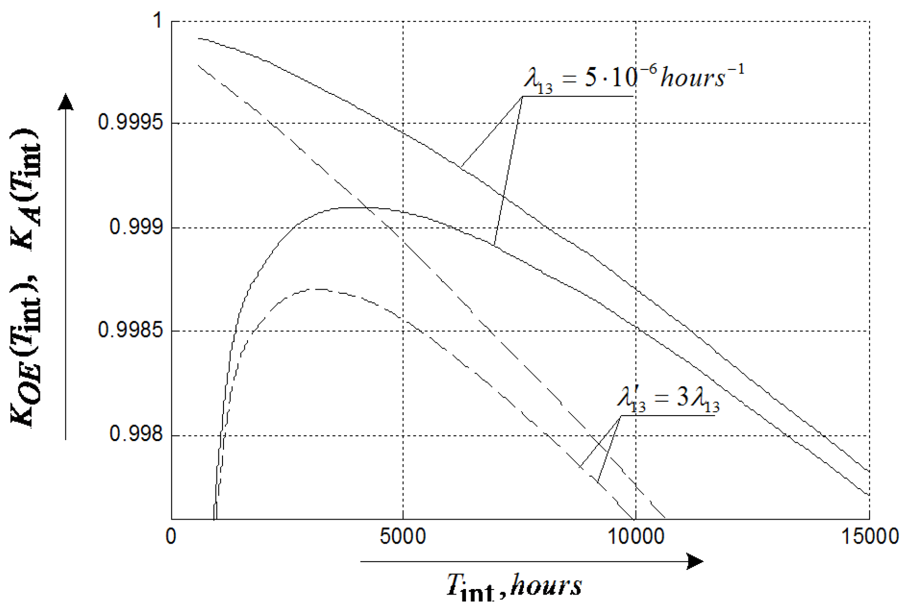

Figure 4. Simulation experiment results

The graphs in Fig. 4 present estimations of the reliability coefficients K_{OE} and K_A obtained with specified accuracy $\varepsilon=1\cdot10^{-5}$. The solid lines refer to the model with the abovementioned values of input parameters. The dashed lines characterize the calculations with three times increased failure rate $\lambda'_{13}=15\cdot10^{-6}$ hours^{-1}. Thus, performing the simulation experiments with developed model allows to obtain estimations of two popular integrated reliability indexes, coefficient of operation efficiency (K_{OE}) and availability coefficient (K_A) depending on the values of input parameters. Moreover, the obtained dependencies nature allows speaking about determination of optimal periodicity of PM and the model could be used in optimization procedures.

CONCLUSION

In this paper a simulation model was proposed for analyzing both the operational process of electronic medium and the estimation of the integrated reliability indexes, coefficient of operation efficiency (K_{OE}) and availability coefficient (K_A). Mathematical formalization of the process is performed using an Embedded Markov chain for semi Markov process model. As the proposed process is a stochastic sequence, then the Monte Carlo method was used in order to simulate its behavior.

The graphical interpretation of the model presents a set of states that are conditioned by a group of operational factors considered in the model. It takes into account the following factors: appearance of sudden, gradual, latent and fictitious failures, human factor of service staff and time parameters of PM. Implementation of simulation experiments provides opportunity to analyze reliability of electronic medium through the coefficients K_{OE} and K_A and to investigate the influence of the factors on reliability indexes.

Subsequent research concerning the simulation model will be devoted to determination of its parameters such as the imitation time T_k and step ΔT in order to obtain maximally adequate results and to implementation the model's verification.

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