Research of Acoustic Wave Generated by Heterogeneous Interface

K. E. Abbakumov, B. Ch. Ee

Saint Petersburg Electrotechnical University "LETI", Saint-Petersburg, Russia
E-mail: ee.boris.eut@gmail.com

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Abstract. We have examined cooperation of plane acoustic wave with cylindrical triggering at interface part where we see a flexible bonding with background medium. To make the flexible bonding at the interface we used stiffness module introduced into boundary conditions. We used scattering cross section to characterize a scattered field. So, we have found that there can be different resonance phenomena in accordance with the size of broken part.

Keywords: plane wave, contact stiffness module, graphite, metal, scattering cross section, finite element method

INTRODUCTION

Cylindrical defect model is popular while research works as to cooperation of real round and extensive defects with ultrasonic wave. This model considers absolutely rigid contact between triggering and background medium. But due to many polished section studies we see that triggering and background medium interface can include sections with partial adhesion fault or full adhesive bonding fault. Cooperating with the similar interface the ultrasonic wave makes a complex diffraction field in bonding with interruptions while transmission of displacement normal component and with sliding of adjoining surfaces while transmission of displacement tangential component [1]. In [2] we show the ability to take into account such interface with the help of interface conditions at linear yawing approach, if there are interruptions while transmission of displacement components and uninterrupted transmission of stress tensor component via stiffness tensor.

MODELING IN COMSOL

Imagine, a plane ultrasonic wave is incident on elastic cylindrical inclusion of a radius, where in the sector of φ dimensions there is adhesive bonding fault, with acoustic parameters ρ_1, c_{l1}, c_{t1} – density, compressional and shear velocity, located at solid isotropic matrix with the following parameters ρ_2, c_{l2}, c_{t2} (Fig. 1).

Let's examine a normal acoustic wave incident which is parallel to Y axis. As the wave is parallel to Y axis so there are not polarized waves in another plane. Cylindrical inclusion (domain 1) with diameter d interact with incident wave and generate scattered and transmitted waves. There is damaged adhesion in the sector (orange line) in the interface between domain 1 and domain 2, which is described as thin elastic layer with normal and tangent contact stiffness (KGN and KGT respectively). It's necessary to study the how contact stiffness affect on the scattered field. This task is solved with the finite element method implemented in the COMSOL software application. Geometrical properties of domain zones are shown in Table 1.

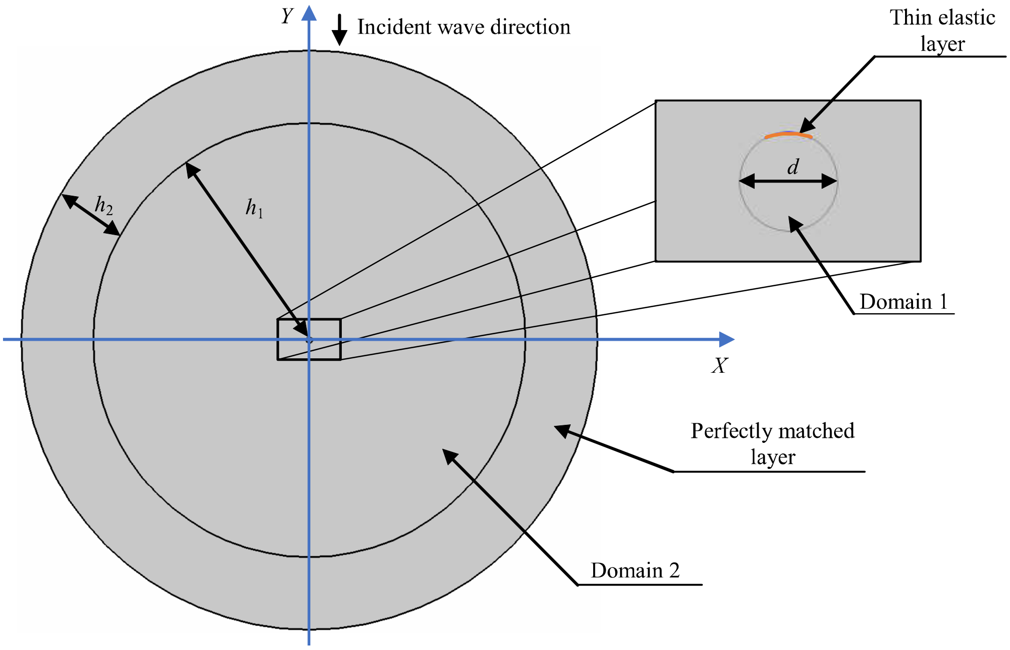

Figure 1. Cylindrical defect in semi-infinite cylindrical matrix

Table 1. Geometrical properties of domain zones

Domain	**Size of domain, mm**
Domain 1	$d = 1$
Domain 2	$h_1 = 6\lambda$
Perfectly matched layer	$h_2 = 2\lambda$

where $\lambda = 2\pi f / c_l$, π – mathematical constant, f – frequency.

The equation describing elastic wave propagation is shown with the wave equation:

$$\Delta u = \frac{1}{c^2}\frac{d^2}{dt^2}, \tag{1}$$

where Δ – Laplace operator; u – displacement; c^2 – phase velocity.

If propagation is harmonic, the equation (1) is changed into the Helmholtz equation:

$$\Delta u + k^2 u = 0,$$

where k – wave number.

If two solids are bonding following boundary conditions are to be taken place in the cylindrical coordinates:

– uninterrupted displacement components:

$$u_r^I = u_r^{II}, \tag{2}$$

$$u_\theta^I = u_\theta^{II}, \tag{3}$$

where u_θ^I, u_r^I, u_θ^{II}, u_r^{II} – components of displacement, upper index denotes domain 1 or 2, lower index denotes component of displacement in cylindrical coordinate;

– uninterrupted stress component:

$$\sigma_{rr}^I = \sigma_{rr}^{II}, \tag{4}$$

$$\sigma_{r\theta}^I = \sigma_{r\theta}^{II}, \tag{5}$$

where σ_{rr}^I, σ_{rr}^{II} – normal component of mechanical stress, upper index denotes domain 1 or 2, lower index denotes component of mechanical stress in cylindrical coordinate; $\sigma_{r\theta}^I$, $\sigma_{r\theta}^{II}$ – tangential component of mechanical stress.

As boundary conditions (2)–(5) in the sector of cylindrical triggering and background medium are changed into [2]:

$$\left.\begin{array}{c} u_r^I + \dfrac{\sigma_{rr}^I}{KGN} = u_r^{II} \\ u_\theta^I + \dfrac{\sigma_{r\theta}^I}{KGT} = u_\theta^{II} \\ \sigma_{rr}^I = \sigma_{rr}^{II} \\ \sigma_{r\theta}^I = \sigma_{r\theta}^{II} \end{array}\right\}, \tag{6}$$

where *KGN*, *KGT* – normal and tangential components of contact stiffness.

Range of values of *KGN*, *KGT* at 10^{12}–10^{17} N/m^3 conforms with the path from free boundary condition to rigid contact [3]. In [4] it is shown the opportunity to make such flexible contact with thin elastic layer with certain values of components of stiffness.

Besides the wave equation and interface conditions we have to model semi-infinite medium. In COMSOL we show semi-infinite medium as a perfectly matched layer embanking the examined area. The perfectly matched layer takes up all the waves entering it and with large losses there is no reflection from the boundaries [5].

To numerical simulation let's choose a steel as a material for domain 2 and perfectly matched layer and a graphite for domain 1 with the following properties as shown in the Table 2 [6, 7].

Table 2. Elastic properties of a chosen materials

Material	Young's modulus *E*, GPa	Shear modulus *G*, GPa	Density, ρ, kg/m^3
steel	5920	3100	7800
graphite	25.5	97	2250

Scattering cross section *SCS* (7) is chosen as a property to be examined. *SCS* is a ratio of the energy flux in the scattered wave to the intensity in the incident wave through the propagation cross line [8]:

$$SCS = \frac{F_s}{I_{inc}}, \tag{7}$$

where $F = -\int \sigma_{ij} \frac{\partial u}{\partial t} n_i dS$; I_{inc} − incident wave intensity; n_i – normal to area S. Additionally, angular distribution of normalized amplitude of displacement in far field is investigated. Displacement amplitude normalized to maximum displacement amplitude at $KN = 10^{12}$ N/m^3.

Results

In the Fig. 2−4 we show scattering cross section *SCS* versus defect wave size $k_l a$ relationship.

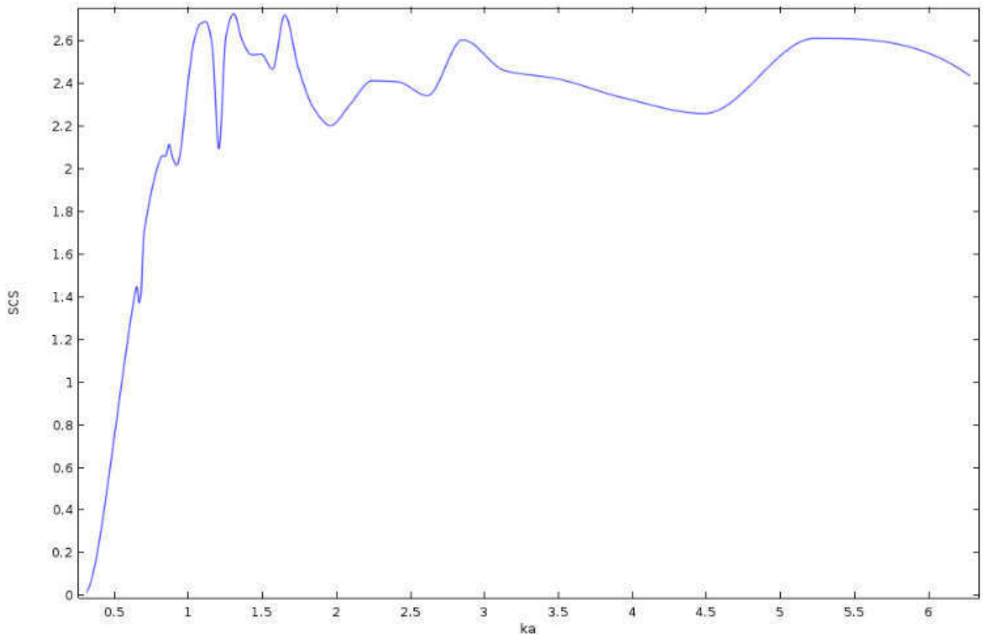

Figure 2. Scattering cross section *SCS* versus defect wave size $k_l a$ relationship if $KN = 10^{15}$ N/m^3

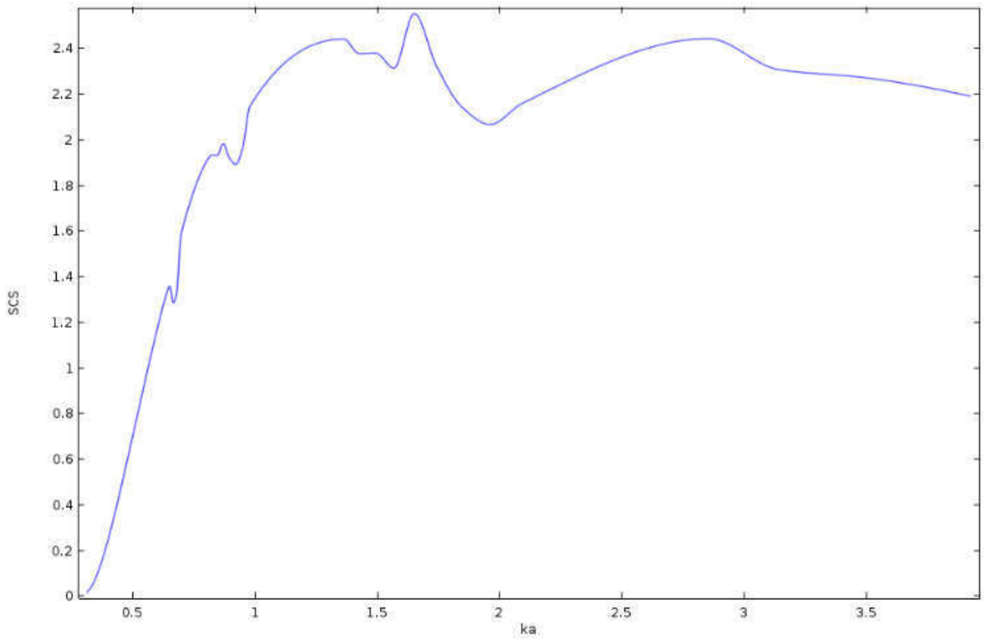

Figure 3. Scattering cross section *SCS* versus defect wave size $k_l a$ relationship if $KN = 10^{13}$ N/m^3

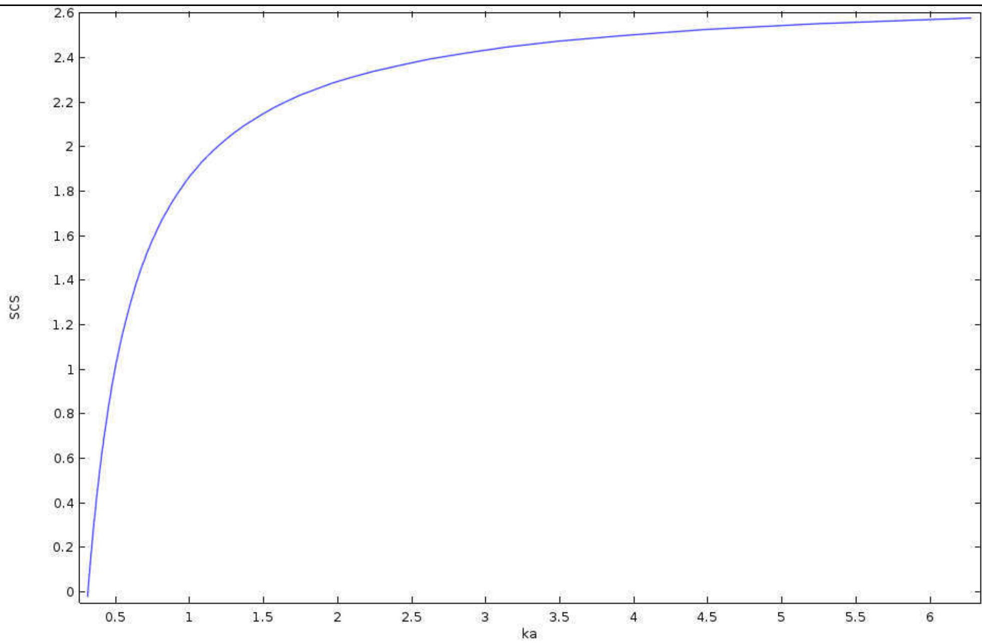

Figure 4. Scattering cross section *SCS* versus defect wave size $k_l a$ relationship if $KN = 10^{12}$ N/m^3

The results of show us that contact rigidity module change causes the change of scattered resulting in resonance phenomena appearance. We see that the change of $KGN = KGT = KN$ in the range from $KN = 10^{12}$ N/m^3 to $KN = 10^{17}$ N/m^3 conforms with the path from a cavity to welded contact. If $KN = 10^{12}$ N/m^3, scattering cross section (Fig. 4) is equal with the results for the cavity in [9].

In Fig. 5–8 the angular distribution of mechanical displacement is shown.

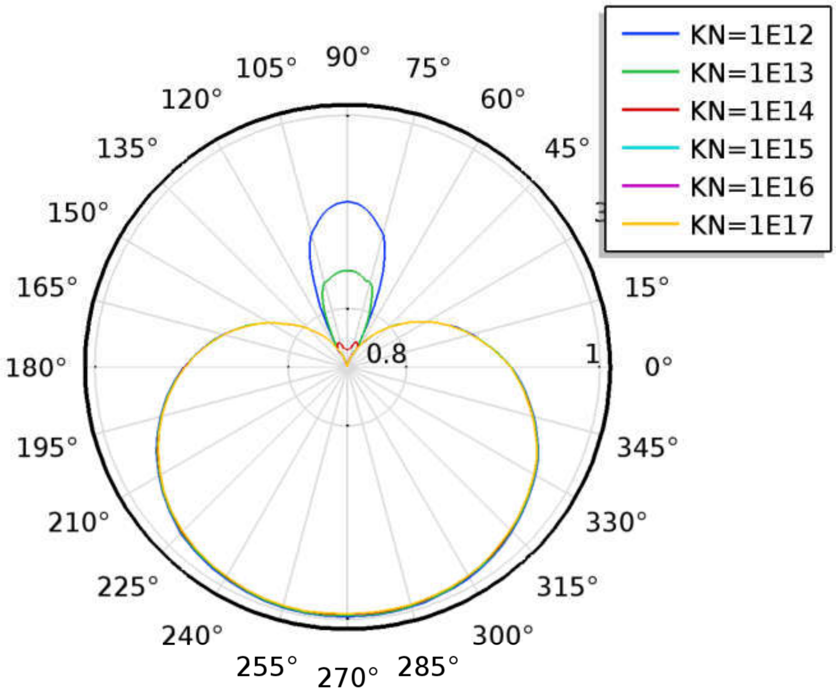

Figure 5. Angular distribution of mechanical displacement
on frequency $f = 2.6 \cdot 10^5$ Hz and various values of *KN*

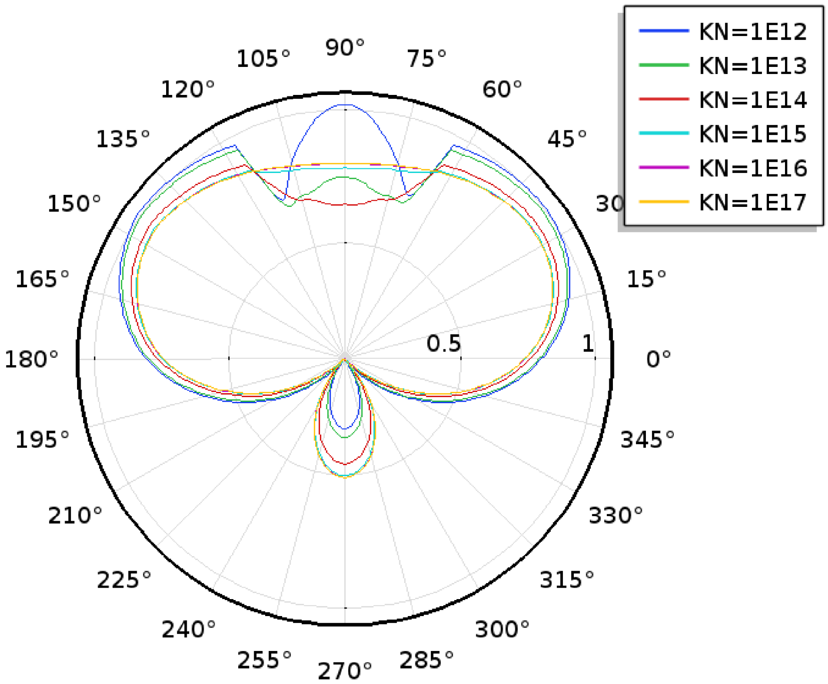

Figure 6. Angular distribution of mechanical displacement on frequency $f = 1 \cdot 10^6$ Hz and various values of *KN*

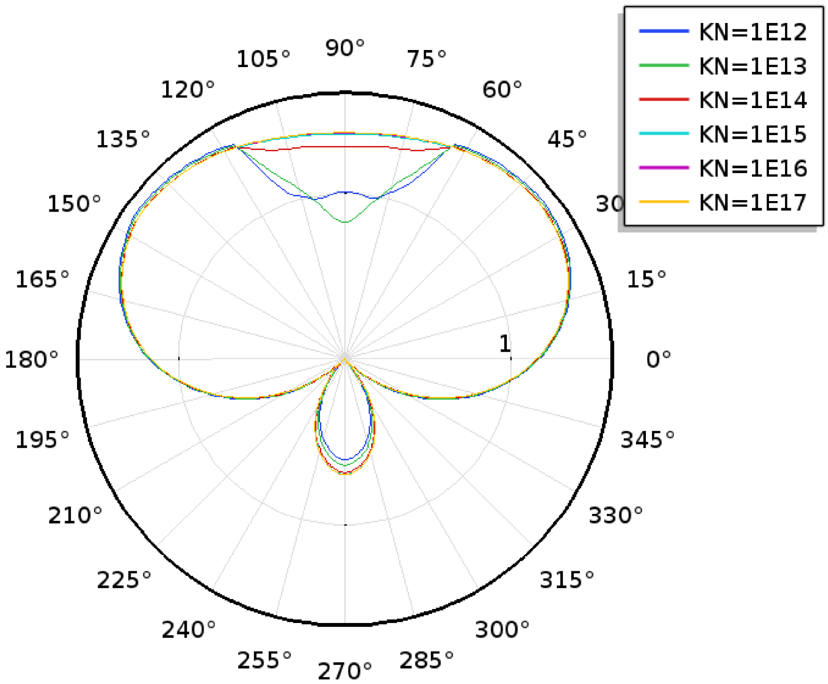

Figure 7. Angular distribution of mechanical displacement on frequency $f = 1.114 \cdot 10^6$ Hz and various values of *KN*

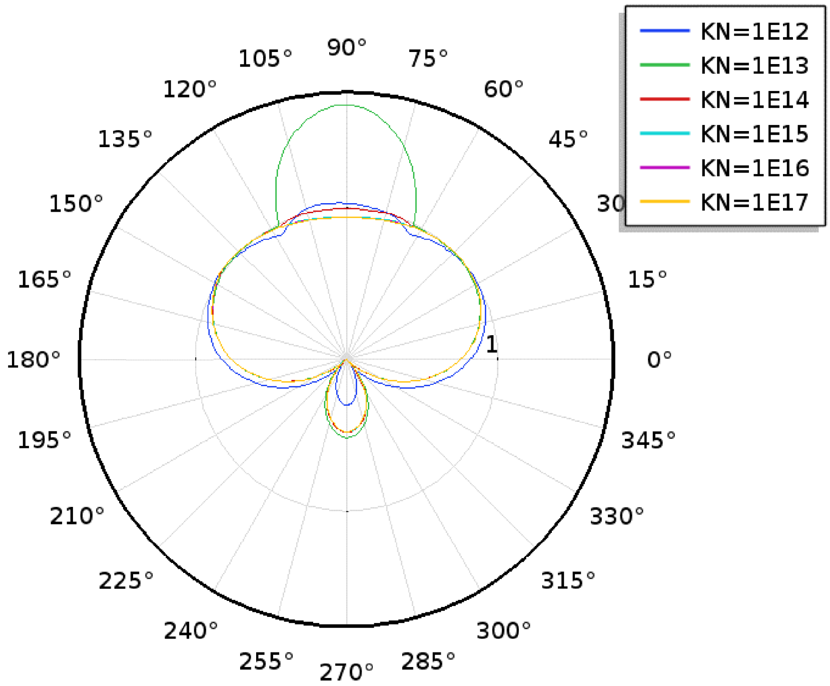

Figure 8. Angular distribution of mechanical displacement on frequency $f = 1.215 \cdot 10^6$ Hz and various values of KN

In Fig. 5–8 we can see that damaged adhesion has different affect on scattered field. For example, in Fig. 5 on the frequency $f = 2.6{\cdot}10^5$ Hz with $KN = 10^{14}$–10^{17} N/m^3 slightly decrease amplitude of displacement in sector, also we can see that the most part of the resulting field is transmitted waves. In Fig. 6 on the frequency $f = 1{\cdot}10^6$ Hz with $KN = 10^{12}$ N/m^3 at the ends of damaged adhesion sector, the amplitude decreases, this may be due to interference on a thin elastic layer. In Fig. 7 on the frequency $f = 1.114{\cdot}10^6$ Hz with $KN = 10^{12}$–10^{14} N/m^3 the displacement amplitude is increase. In Fig. 8 on the frequency $f = 1.215{\cdot}10^6$ Hz with $KN = 10^{13}$ N/m^3 the displacement amplitude is slightly increase, it can be related with resonant scattering or thin elastic layer can be as a quarter-wave matching layer.

Conclusion

Scattering properties of cylindrical heterogeneity are related to the contact quality between the heterogeneity and background medium very much. So damaged adhesion has a large effect on information signals against heterogeneities. It should be taken into account at nondestructive testing, since it can cause both the undetection of the defect or a valid defect can be taken as invalid.

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