Minicopter Nonlinear Control with the Existence of Disturbances

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Abstract. This paper is focusing on studying the nature of equations of motion of a six degree of freedom of a Minicopter. Choosing a Minicopter is challenging in the field of control because it is a highly nonlinear, multivariable, and underactuated system, in addition to its advantages such as high maneuverability and stationary flight. Underactuated systems, defined as a mechanical system in which the dimension of the configuration space exceeds that of the control input space, that is, with fewer control inputs than the degrees of freedom. Modeling of such a system is not a trivial problem due to the coupled dynamics of the aerial vehicle. The dynamic model is formulated using the Newton - Euler method for translational and rotational dynamics, and the contribution of this work is deriving an accurate and detailed mathematical model. Then disturbances that represent outdoors environment were added to be used in the simulation, the topic, which was unmentioned before in most of the literature. The kinematics and dynamics were studied then equations of motion were explained. The state space model was derived with the existence of disturbances in the earth frame. An application was conducted by LabVIEW simulation program using Runge-Kutta 2 method. Correlations were analyzed on all parameters of motion equations; four slide mode controllers were implemented to stabilize the altitude and attitude. Finally, Lyapunov stability was presented.

Keywords: Minicopter, disturbances, Lyapunov stability, UAV control

1. INTRODUCTION

Nowadays, mini-drones invaded several application domains [1–3]. The control of aerial robot such as Minicopter requires dynamics in order to account for gravity effects and aerodynamic forces [1–2]. These aerial vehicles have high maneuverability and stationary flight [4–5]. In this work equations of motion were concluded of the whole system using the Newton-Euler formulation for translational and rotational dynamics of a rigid body [1, 6]. This paper is focusing on studying the nature of equations like nonlinearity and coupled variables, then adding disturbances that were presented as an environment for outdoors simulation [3, 7–9], which is unmentioned in most of the literature. The structure of the paper is as follows: describing kinematics and dynamics then explain equations of motion.

2. Reference Systems for the UAV Minicopter

In order to describe the Minicopter motion only two reference systems are necessary: earth inertial frame (E-frame) and body-fixed frame (B-frame). An Inertial frame is a system that uses the North, East, and Down (NED) coordinates and the origin of this reference system is fixed in one point located on the earth surface as shown in Figure 1, and the (X, Y, Z) axes are directed to the North, East, and Down, respectively. The mobile frame (XB, YB, ZB) is the body fixed frame that is centered in the Minicopter center of gravity and oriented as shown in Figure 1. The angular position of the body frame with respect to the inertial one is usually defined by means of the Euler angles: roll ϕ, pitch θ, and yaw ψ. As the vector: $\sigma = [\phi \quad \theta \quad \psi]^T$, ϕ and $\theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$; $\psi \in]-\pi, \pi[$. The inertial frame position of the vehicle is given by vector $\xi = [x \quad y \quad z]^T$ [1–2, 5–6]. The transformation from the body frame to the inertial frame is realized by using the well-known rotation matrix C_b^n [1, 4, 6]:

$$C_b^n = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & c\phi s\theta s\psi - s\phi\, c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix},$$

which is orthogonal, and $c\theta$ equivalent to $\cos\theta$ also $s\theta$ means $\sin\theta$, while the transformation matrix for angular velocities from the body frame to the inertial one is S [3, 5].

$$S = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix},$$

where $\dot{\sigma} = S.\Omega$, $\dot{\xi} = C_b^n.V$, the angular velocity Ω is defined by the vector $\Omega = [p\,q\,r]^T$, and the linear velocity is defined by the vector $V = [u\,v\,w]^T$ in the body frame [1–6].

3. Aerodynamic Forces and Moments in Axial Flight

The UAV Minicopter systems are quite complex; their movements are governed by several effects either mechanical or aerodynamic. Our aim is to provide the mathematical equations driving the dynamical behavior of the Minicopter by means of a generalization of the Quadcopter model presented in [1, 5–7]. The motion of a rigid body can be decomposed into the translational and rotational components. Therefore, in order to describe the dynamics of the Minicopter, assumed to be a rigid body, the Newton-Euler equations, that govern linear and angular motion are used. In order to get equations of motion of entire system, the following assumptions have been made:

- The Minicopter is a rigid body.
- The Minicopter has a symmetrical structure.

Therefore, the following equations are obtained:

$$\begin{bmatrix} m I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\Omega} \end{bmatrix} + \begin{bmatrix} \Omega\times(mV) \\ \Omega\times(J\Omega) \end{bmatrix} = \begin{bmatrix} \Sigma F \\ \Sigma M \end{bmatrix}.$$

3.1. Force Analysis

a. Thrust Force

The main force affecting the aircraft movement is the thrust force resulting from the motors and propellers that leads to raise the aircraft in the air. The model consists of 6 motors, and according to the suggested engineering model, the motors in the model are parallel and perpendicular on the aircraft surface, so we conclude that the total thrust force vector of the aircraft is T and it is the sum of the propellers thrust force vectors $\sum_{i=1}^{6} T_i$. The main rotor thrust T orientation is expressed in terms of the lateral and longitudinal cyclic tilt angles a and b (Fig. 1) [4].

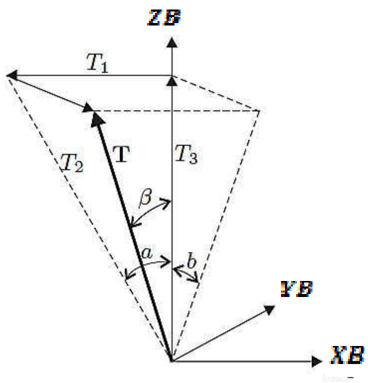

Figure 1. Thrust vector

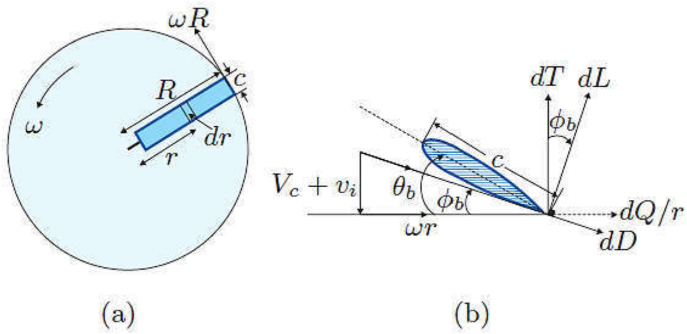

Figure 2. Force components on blade in vertical flight

Then, the main rotor thrust may be expressed as a vector in the body fixed fame as: $T = \frac{1}{\sqrt{1-\sin^2 a \,\sin^2 b}} \begin{bmatrix} \sin a \,\cos b \\ \cos a \,\sin b \\ \cos a \,\cos b \end{bmatrix} .|T|$. By applying the small-angle assumption T, the total lift of the main Minicopter tilted in comparison to the motor shaft can be rewritten as: $T = |T|.[a \quad b \quad 1]^T$. The aerodynamic forces and moments are derived using a momentum combination; the left force depends on the angular velocity ω and the geometric dimensions of propellers as seen in a top view of the rotor disc in Fig. 2-a where blade rotation is counterclockwise with angular velocity ω. The blade radius is R; the tip speed therefore is ωR. An elementary blade section is taken at radius r of chord length c and span-wise width d_r. Forces on the blade section are shown in Fig. 2-b. The flow seen by the section has velocity components: ωr in the disc plane, $(V_c + v_i)$ in the axial, and induced velocities perpendicular to it. The angle θ_b denotes the blade pitch angle, and ϕ_b is the inflow angle. In addition, dT, dQ, dL, and dD are the elementary thrust, torque, lift, and drag forces respectively. The thrust and torque, are [5]: $|T_i| = \rho C_T A R^2 \omega_i^2$, $|Q_i| = \rho C_Q A R^3 \omega_i^2$, where blade rotation is with angular velocity ω, the blade radius is R, C_T and C_Q are the thrust and torque coefficients respectively, ρ is the air density, and A the disc area. The thrust and torque coefficients can be written as:

$$C_T = \frac{1}{4}\sigma C_{L\alpha}\left[\frac{2\theta_b}{3} - (\gamma_c + \gamma_i)\right],\; C_Q = \frac{1}{2}\sigma[C_{L\alpha}(\gamma_c + \gamma_i)\left\{\frac{\theta_b}{3} - \frac{\gamma_c + \gamma_i}{2}\right\} + \frac{C_D}{4},$$

where σ is the rotor solidity, $C_{L\alpha}$ is the lift slope coefficient, C_D is the drag coefficient γ_c, and γ_i are the inflow factors [5]. Finally, the total force of thrust generated by the six propellers in the earth frame is defined as:

$$F_{TI} = C_b^n \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^6 |T_i| \end{bmatrix}_B = \begin{bmatrix} (c\phi\, c\psi\, s\theta + s\phi\, s\psi) \sum_{i=1}^6 |T_i| \\ (c\phi\, s\theta\, s\psi - s\phi\, c\psi) \sum_{i=1}^6 |T_i| \\ (c\phi\, c\theta) \sum_{i=1}^6 |T_i| \end{bmatrix}_E .$$

b. Drag Force

It is the opposing force to the travelling of the Minicopter in air, which is resulting from the aerodynamic friction, air density, velocity, and can be expressed by the following equation at the earth's frame: $F_{AI} = K_{TI}.\dot{\xi}$, where K_{TI} is a diagonal matrix related to the aerodynamic friction constant k_t [4][5].

c. Gravitational Force

The gravity force is directed toward the center of the earth and the relation of gravity force in the earth frame by [1][2]: $F_{GI} = m[0 \;\; 0 \;\; g]_E^T$.

d. Disturbance Force

Other forces like Coriolis force from the earth, wind, and Euler forces are considered as **disturbances, summarized as F_{DI} in the Earth frame: $F_{DI}=[F_{dIx} \;\; F_{dIy} \;\; F_{dIz}]_E^T$. Therefore, the** equations of motion that govern the translational motion with respect to the Earth frame are:

$$\Sigma M = M_T - M_{AI} + M_{gyro} + M_{DI} = J.\dot{\Omega} + \Omega \times (m.\Omega)$$

$$\begin{bmatrix} M_p \\ M_q \\ M_r \end{bmatrix}_E - \begin{bmatrix} k_r \dot{\phi}^2 \\ k_r \dot{\theta}^2 \\ k_r \dot{\psi}^2 \end{bmatrix}_E + \begin{bmatrix} 0 \\ 0 \\ J_r \dot{\omega}_r \end{bmatrix}_E + \begin{bmatrix} 0 \\ 0 \\ J_r \dot{\omega}_r \end{bmatrix}_E + \begin{bmatrix} M_{dI\phi} \\ M_{dI\theta} \\ M_{dI\psi} \end{bmatrix}_E = J \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}_E ,$$

where $\Omega \times (m.\Omega)$ has a small effect and approximately equal to zero. Therefore, the equations after some simplifications will be:

$$\left\langle \begin{array}{l|l} \ddot{x} = U_x - \dfrac{k_t}{m}\dot{x} + \dfrac{F_{dIx}}{m} & U_x = (c\phi c\psi s\theta + s\phi s\psi) u_T / m = a_x(\phi,\theta,\psi) \sum_{i=1}^6 \omega_i^2 \\ \ddot{y} = U_y - \dfrac{k_t}{m}\dot{y} + \dfrac{F_{dIy}}{m} & U_y = (c\phi s\theta s\psi - s\phi c\psi) u_T / m = a_y(\phi,\theta,\psi) \sum_{i=1}^6 \omega_i^2 \\ & U_z = (c\phi c\theta) u_T / m = a_z(\phi,\theta) \sum_{i=1}^6 \omega_i^2 \\ \ddot{z} = U_z - \dfrac{k_t}{m}\dot{z} - g + \dfrac{F_{dIz}}{m} & u_T = \sum_{i=1}^6 |T_i| = \rho C_T A R^2 \sum_{i=1}^6 \omega_i^2 \\ & a = \dfrac{k_t}{m} \to 0 \end{array} \right\rangle .$$

Therefore, the final equations with respect to the earth frame are:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -a\dot{x} + U_x \\ -a\dot{y} + U_y \\ -a\dot{z} + U_z - g \end{bmatrix} + \begin{bmatrix} \frac{F_{dIx}}{m} \\ \frac{F_{dIy}}{m} \\ \frac{F_{dIz}}{m} \end{bmatrix} = \begin{bmatrix} U_x \\ U_y \\ U_z - g \end{bmatrix} + \begin{bmatrix} \frac{F_{dIx}}{m} \\ \frac{F_{dIy}}{m} \\ \frac{F_{dIz}}{m} \end{bmatrix}.$$

3.2. Moments Analysis

The aircraft is affected by several types of moments: the thrust moment resulting from the motors, the motors inertia moment, the aerodynamic moment, and the disturbances moment. Supposing, the inertia matrix of the aircraft is J, the structure of the aircraft is symmetric, and the inertia matrix is of the following form: $J = [J_{xx}\ 0\ 0;\ 0\ J_{yy}\ 0;\ 0\ 0\ J_{zz}]^T$; $J \in R_{3\times3}$ [5], therefore, the moments acting on the center of the aircraft can be analyzed in the following:

a. Propeller Moments

The moment M_{thrust} is part of the external moments, that described by the propeller thrust $\sum_{i=1}^{6} T_i$ generated by the propellers, and the distance l from CG to the center of the propeller. The attitude of the vehicle in the air, i.e., Euler angles $\sigma = [\phi\ \theta\ \psi]^T$ change, by controlling the angular velocity of the motors. This means, there is a different thrust moment over time $M_T = [M_p\ M_q\ M_r]^T$, where M_p, M_q, M_r are the moments about the axes X_B, Y_B, Z_B in the body frame [1, 3, 4], noticing that the torque vectors across each other are in the same direction, and the motors that are closest to each other have torque vectors in opposite directions as in Fig. 3.

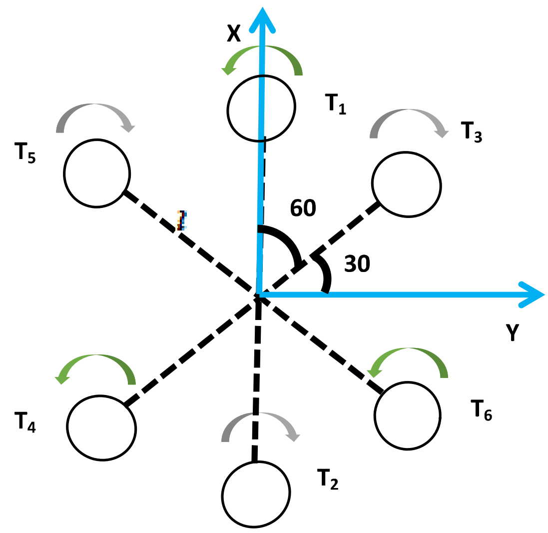

Figure 3. Minicopter architecture

This is the design requirement to keep the Minicopter from spinning out of control [1, 5, 7]. The moments generated by motors as shown in Figure 3, can be explained as follows:

$$M_T = \begin{bmatrix} \frac{\sqrt{3}}{2} l\left(|T_3| - |T_4| - |T_5| + |T_6|\right) \\ \frac{1}{2} l\left(|T_3| - |T_4| + |T_5| - |T_6| + 2|T_1| - 2|T_2|\right) \\ \rho C_Q A R^3 \left(\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2\right. \end{bmatrix}.$$

b. The aerodynamic moment

It is the moment resulting from the aerodynamic friction in air and is proportional to the torque around the axes, and it is expressed by the following equation: $M_{AI} = K_{RI}.\Omega^2 = K_{RI}\left[\dot{\phi}^2 \ \ \dot{\theta}^2 \ \ \dot{\psi}^2\right]^T$, where K_{RI} is a diagonal matrix related to the rotational aerodynamic friction constant by the parameter k_r [4–5].

c. Disturbance moment

It is the total of the disturbances affecting the torque around the aircraft axes resulting from disturbances in the motors movement, the wind, and the load in the aircraft, expressed as: $M_{DI} = \left[M_{dI\phi} \ \ M_{dI\theta} \ \ M_{dI\psi}\right]^T$.

d. Propeller Gyroscopic effect

The rotation of the propellers produces a gyroscopic effect: $M_{gyro} = \left[-J_r\dot{\theta}\omega_r \quad J_r\dot{\phi}\omega_r \quad 0\right]^T$ [3], where J_r is the rotational inertia of the propeller $\left[NmS^2\right]$, and ω_r $[rad / S]$ is the overall propeller speed: $\omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4 - \omega_5 + \omega_6$.

e. Yaw counter moment

Differences in rotational acceleration of the propellers produces a yaw inertial counter moment as follow: $M_{counter} = [0 \quad 0 \quad J_r\dot{\omega}_r]^T$ [3]. Therefore, the equations of motion that govern the rotational motion with respect to the body frame are:

$$\Sigma M = M_T - M_{AI} + M_{gyro} + M_{counter} + M_{DI} = J.\dot{\Omega} + \Omega \times (m.\Omega)$$

$$\begin{bmatrix} M_p \\ M_q \\ M_r \end{bmatrix}_E - \begin{bmatrix} k_r\dot{\phi}^2 \\ k_r\dot{\theta}^2 \\ k_r\dot{\psi}^2 \end{bmatrix}_E + \begin{bmatrix} -J_r\dot{\theta}\omega_r \\ J_r\dot{\phi}\omega_r \\ 0 \end{bmatrix}_E + \begin{bmatrix} 0 \\ 0 \\ J_r\dot{\omega}_r \end{bmatrix}_E + \begin{bmatrix} M_{dI\phi} \\ M_{dI\theta} \\ M_{dI\psi} \end{bmatrix}_E = J\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}_E + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times J\begin{bmatrix} p \\ q \\ r \end{bmatrix},$$

$$\left\langle \begin{array}{l|c} \dot{p} = \dfrac{M_p}{J_x} + qr\dfrac{(J_y - J_z)}{J_x} - \dfrac{k_r}{J_x}p - \dfrac{J_r}{J_x}\dot{\theta}\omega_r + \dfrac{M_{dI\phi}}{J_x} & Transformation \quad Equation \\ \dot{q} = \dfrac{M_q}{J_y} + pr\dfrac{(J_z - J_x)}{J_y} - \dfrac{k_r}{J_y}q + \dfrac{J_r}{J_y}\dot{\phi}\omega_r + \dfrac{M_{dI\theta}}{J_y} & \dot{\sigma} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_E = S.\begin{bmatrix} p \\ q \\ r \end{bmatrix}_B \\ \dot{r} = \dfrac{M_r}{J_z} + pq\dfrac{(J_x - J_y)}{J_z} - \dfrac{k_r}{J_z}r + \dfrac{J_r}{J_z}\dot{\omega}_r + \dfrac{M_{dI\psi}}{J_z} & \end{array} \right\rangle.$$

Transformation matrix $S \to I$ when the hexa-copter tends to the stable point, therefore the equations of angular rate will be related to the Earth frame, in addition to some assumptions: $\dfrac{J_r}{J_x} = \dfrac{J_r}{J_y} = \dfrac{J_r}{J_z} \to 0$ is a small effect around zero, and $J_x = J_y$. Then the equations will be as follows:

$$\left\langle \begin{array}{l|c} & U_p = \dfrac{M_p}{J_x} = \dfrac{\sqrt{3}\rho l C_T A R^2}{2J_x}\left(\omega_3^2 + \omega_6^2 - \omega_4^2 - \omega_5^2\right) \\ & U_p = a_\phi\left(\omega_3^2 + \omega_6^2 - \omega_4^2 - \omega_5^2\right) \\ \ddot{\phi} = U_p + b_1\dot{\theta}\dot{\psi} + c_1\dot{\phi}^2 + \dfrac{M_{dI\phi}}{J_x} & U_q = \dfrac{M_q}{J_y} = \dfrac{\rho l C_T A R^2}{2J_y}\left(\omega_3^2 + \omega_5^2 + 2\omega_1^2 - \omega_4^2 - \omega_6^2 - 2\omega_2^2\right) \\ \ddot{\theta} = U_q + b_2\dot{\phi}\dot{\psi} + c_2\dot{\theta}^2 + \dfrac{M_{dI\theta}}{J_y} & U_q = a_\theta\left(\omega_3^2 + \omega_5^2 + 2\omega_1^2 - \omega_4^2 - \omega_6^2 - 2\omega_2^2\right) \\ \ddot{\psi} = U_r + c_3\dot{\psi}^2 + \dfrac{M_{dI\psi}}{J_z} & U_r = \dfrac{M_r}{J_z} = \dfrac{\rho l C_Q A R^3}{2J_z}\left(\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2\right) \\ & U_r = a_\psi\left(\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2\right) \\ & b_1 = \dfrac{J_y - J_z}{J_x}, b_2 = \dfrac{J_z - J_x}{J_y}, c_1 = -\dfrac{k_r}{J_x}, c_2 = -\dfrac{k_r}{J_y}, c_3 = -\dfrac{k_r}{J_z} \end{array} \right\rangle.$$

Figure 4 shows the diagram of the control system model using LabVIEW application that contains the programming of the equations of motion. These equations require entering the input variables and physical characteristics for getting the state variables.

4. State Space Model

The dynamics model presented in the translational and rotational equation set can be rewritten in the state-space form as:

$$\dot{X} = f(X, U) + \delta.$$

where δ is the disturbances, and the $X \in \mathbf{R}^{12}$ is the vector of state variables given as follow:

$$X = [x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z} \quad \phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi}]^T.$$

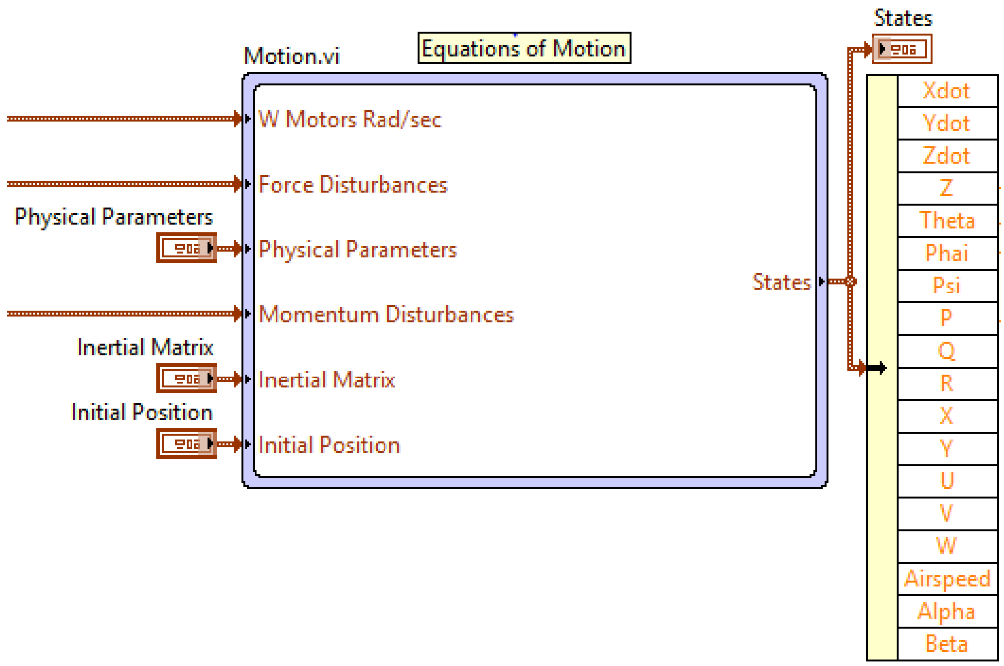

Figure 4. Mathematical model designed in LabVIEW

Then we can conclude to the final equations of the system in state space, which governs the transitional and rotational of the hexa-copter with respect to the Earth Frame, as follows:

$$\begin{pmatrix}\dot{x}_2\\ \dot{x}_4\\ \dot{x}_6\end{pmatrix}=\begin{pmatrix}-a & 0 & 0\\ 0 & -a & 0\\ 0 & 0 & -a\end{pmatrix}\begin{pmatrix}x_2\\ x_4\\ x_6\end{pmatrix}+\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}U_x\\ U_y\\ U_z\end{pmatrix}+\begin{pmatrix}F_{dIx}/m\\ F_{dIy}/m\\ \dfrac{F_{dIz}}{m}-g\end{pmatrix}=\begin{pmatrix}U_x\\ U_y\\ U_z\end{pmatrix}+\begin{pmatrix}F_{dIx}/m\\ F_{dIy}/m\\ \dfrac{F_{dIz}}{m}-g\end{pmatrix};$$

$$\begin{pmatrix}\dot{x}_8\\ \dot{x}_{10}\\ \dot{x}_{12}\end{pmatrix}=\begin{pmatrix}b_1x_{10}x_{12}+c_1x_8^2\\ b_2x_8x_{12}+c_2x_{10}^2\\ b_3x_8x_{10}+c_3x_{12}^2\end{pmatrix}+\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}U_p\\ U_q\\ U_r\end{pmatrix}+\begin{pmatrix}M_{dI\phi}/J_x\\ M_{dI\theta}/J_x\\ M_{dI\psi}/J_x\end{pmatrix}.$$

The suggested real and complex dynamics mathematical model were derived as shown in the equations before. Basically, this model was consisting of 6 equations and characterized by nonlinearity, time-variance, and coupling among the system variables where any change in the input variables leads to changes in most of the output variables. The control mechanism consists of two-level control, the first level controls the horizontal motion using the input vector $U_1=[U_x \quad U_y]^T$, while the next level controls the attitude and altitude motion using the control vector $U_2=[U_p \quad U_q \quad U_r \quad U_z]^T$, and the command control of the Minicopter is vector $D=[x_d \quad y_d \quad z_d \quad \psi_d]^T$, which gives the position and rotation in the space. Furthermore, the horizontal control (x, y) depends on the angles ϕ, θ of the aircraft. With respect to the control input vector $U=[U_x \quad U_y \quad U_z \quad U_p \quad U_q \quad U_r]^T$, it is clear that the rotational subsystem is fully-actuated, it is only dependent on the rotational state variables x_7 to x_{12}, while the translational subsystem is under-actuated as it is dependent on both the translational state variables x_1 to x_6 and the rotational ones x_7 to x_{12}. The control strategy is through controlling the motors speed variables $\omega_1, \omega_2 \ldots \omega_6$ by a defined style explained as follows:

1. There are 3 movements that describe all possible combinations of attitude: Roll (rotation around the *X* axis by angle ϕ), Pitch (rotation around the *Y* axis by angle θ), and Yaw (rotation around the *Z* axis by angle ψ). The roll control is obtained by changing the velocity of motors 3, 4, 5, 6, and this movement is called lateral motion. Then the pitch control is obtained by changing the velocity of all motors, resulting in the longitudinal motion. Finally, the yaw control is obtained by changing the velocity of all motors.

2. The change of motors speed for attitude control should be fixed and based on differential control strategy as seen in Fig. 3 and equations of moments, i.e., the Pitch control around axis *Y* is obtained by changing the torques around this axis by increasing (T_1, T_3, T_5) and decreasing another side (T_2, T_4, T_6) using the following equation:

$$Pitch\_control \Leftrightarrow l \times \cos 60 \times \left(|T_3| - |T_4| + |T_5| - |T_6|\right) + |T_1| - |T_2|.$$

The roll control is obtained as follows:

$$roll\_control \Leftrightarrow l \times \cos 30 \times \left(|T_3| - |T_4| - |T_5| + |T_6|\right).$$

While the yaw control is based on the torque difference between the neighboring motors:

$$yaw\_control \Leftrightarrow Q_1 + Q_4 + Q_6 - Q_2 - Q_3 - Q_5.$$

3. Altitude control is obtained by changing all motors’ velocity with a fixed change. This is based on the force equations in Z component, noticing that the thrust is equivalent to the square of the motors angular velocities. To increase the altitude, all motors velocities must be increased, and vice versa. The equation that governs the altitude is:

$$altitude\_control \Leftrightarrow \sum_{i=1}^{6} |T_i|.$$

From the control problem based on Fig. 5 and its angular speed correction, which govern the attitude and altitude in space, the artificial vector $U_2 = [U_p \quad U_q \quad U_r \quad U_z]^T$ can be found. This simplifies the control of the system in Pitch, Roll, Yaw and altitude movements instead of using real motors’ velocities vector ω [5]. Now we can put the equations that connect between artificial and real input vectors as follows:

$$\begin{bmatrix} U_P \\ U_q \\ U_r \\ U_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & +a_\phi & -a_\phi & -a_\phi & +a_\phi \\ 2a_\theta & -2a_\theta & +a_\theta & -a_\theta & +a_\theta & -a_\theta \\ +a_\psi & -a_\psi & -a_\psi & +a_\psi & -a_\psi & +a_\psi \\ +a_z & +a_z & +a_z & +a_z & +a_z & +a_z \end{bmatrix} \cdot \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \\ \omega_5^2 \\ \omega_6^2 \end{bmatrix}.$$

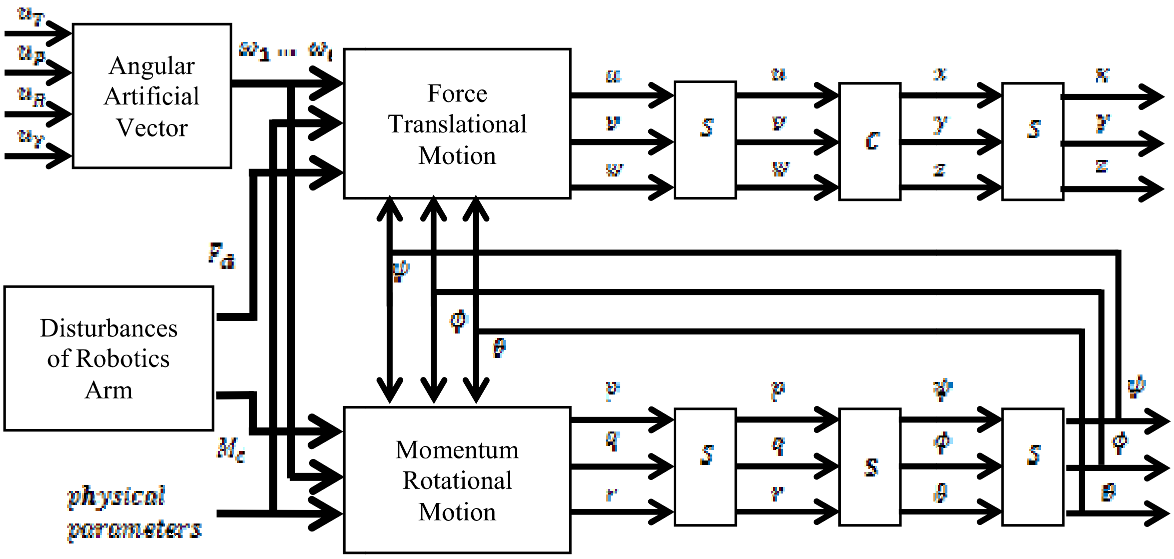

Figure 5. Block diagram of a dynamical system for Minicopter

5. Slide Mode Control and Lyapunov Stability

The sliding mode controller designing procedure is exerted in two steps. Firstly, the choice of the sliding surface (S) is produced according to the tracking error; in the second step a Lyapunov function is considered verifying the necessary condition for the stability in Lyapunov law. The sliding mode control of the state variables estimated dynamics is presented by establishing the statement of the control input. The sliding surfaces are described as follows [4, 8–9]:

$$S_x = e_2 + \lambda_1 e_1, \quad S_y = e_4 + \lambda_2 e_3, \quad S_z = e_6 + \lambda_3 e_5,$$
$$S_p = e_8 + \lambda_4 e_7, \quad S_q = e_{10} + \lambda_5 e_9, \quad S_r = e_{12} + \lambda_6 e_{11}$$

such that $\lambda_i > 0$ and $e_i = x_{id} - x_i$, $i \in [1,11]$, $e_{i+1} = \dot{e}_i$. The following Lyapunov function is chosen: $V(S_x) = \frac{1}{2} S_x^2$, then the necessary sliding condition is verified and Lyapunov stability is guaranteed. The chosen law for the attractive surface is the time derivative of $V(S_x)$ satisfying $(S_x \dot{S}_x < 0)$ [8, 9] where:

$$\dot{S}_x = -k_1 sign(S_x) = \dot{x}_{2d} - \dot{x}_2 + \lambda_1 \dot{e}_1 = \ddot{x}_{1d} + a x_2 - U_x - \frac{F_{dIx}}{m} + \lambda_1 \dot{e}_1$$

$$U_x = -k_1 sign(S_x) + a x_2 - \frac{F_{dIx}}{m} + \ddot{x}_d + \lambda_1 e_2$$

The same steps are followed to extract U_y, U_z, U_p, U_q, U_r:

$$U_y = -k_2 \operatorname{sign}(S_y) + a x_4 - \frac{F_{dIy}}{m} + \ddot{y}_d + \lambda_2 e_4,$$

$$U_z = -k_3 \operatorname{sign}(S_z) + a x_6 - \frac{F_{dIz}}{m} + g + \ddot{z}_d + \lambda_3 e_6,$$

$$U_p = -k_4 \operatorname{sign}(S_p) - b_1 x_{10} x_{12} - c_1 x_8^2 - \frac{M_{dI\phi}}{j_x} + \ddot{p}_d + \lambda_4 e_8,$$

$$U_q = -k_5 \operatorname{sign}(S_q) - b_2 x_8 x_{12} - c_2 x_{10}^2 - \frac{M_{dI\theta}}{j_y} + \ddot{q}_d + \lambda_5 e_{10},$$

$$U_r = -k_6 \operatorname{sign}(S_r) - b_3 x_8 x_{10} - c_3 x_{12}^2 - \frac{M_{dI\psi}}{j_x} + \ddot{r}_d + \lambda_6 e_{12}.$$

6. Experiments and Results

The system's parameters that are used in the simulation of the model, are listed in table 1. Real and complex dynamics model was taken into account, which has addressed the nonlinearity, time variance, under-actuation, and disturbances with respect to the earth frame. An application was conducted by LabVIEW simulation program using Runge-Kutta 2 method with fixed step 0.05 (sec). Correlations were analyzed on all parameters of motion equations; four slide mode controllers were implemented to stabilize the altitude and attitude as shown in figures (6) to (9) that we may notice the stability and disturbances free as possible in our response with multiple set points taken into account. The tuning process achieved after multi attempts of experiments. Our scenario is the hovering flight at altitude 10 meters in the air.

Table 1. Parameters used in the simulation

m=4 kg	**g=9.806 m/s^2**	**l=0.36 m**
I_x, I_y = 3.8e^{-3} N.m.s^2/rad	R = 0.15 m	k_t = 4.8e^{-2} N.s/m
I_z = 7.1e^{-3} N.m.s^2/rad	A = 0.071 m^2	k_r = 6.4e^{-4} N.m.s/rad
C_T = 0.01458	C_Q = 1.037e^{-3}	ρ = 1.293 kg/m^3

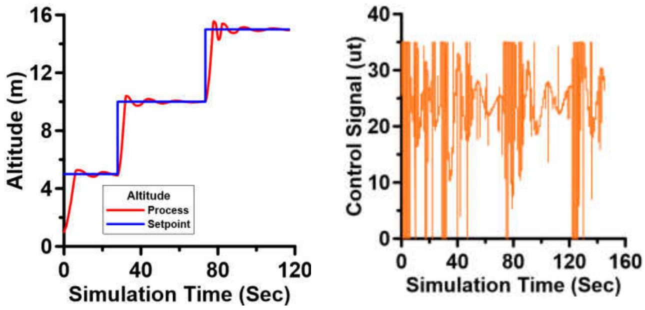

Figure 6. Stability response of altitude and control signal

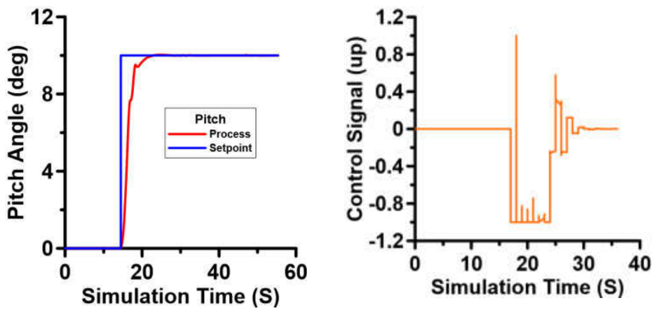

Figure 7. Stability response of pitch angle and control signal

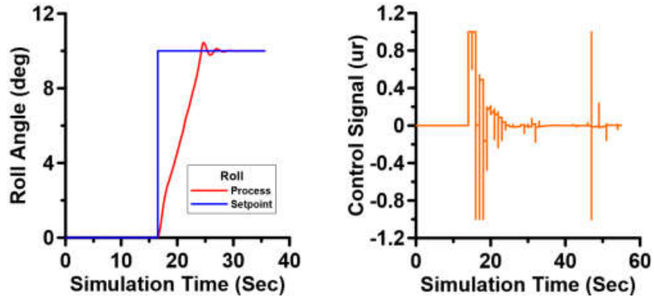

Figure 8. Stability response of roll angle and control signal

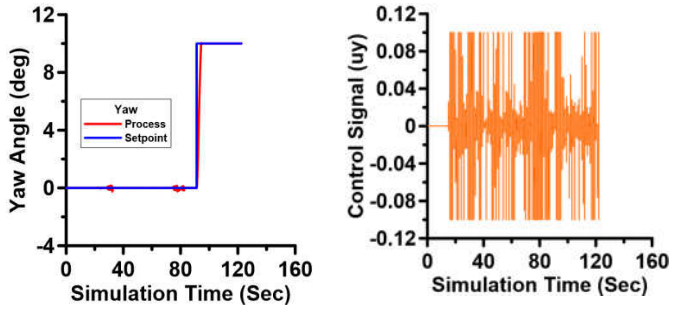

Figure 9. Stability response of yaw angle and control signal

7. The Conclusion

In this work, controllers were designed and were tuned in order to control a Minicopter, a special type of UAV. In this paper, real and complex dynamics model was taken into account, which has addressed the nonlinearity, time variance, under-actuation, and disturbances with respect to the earth frame. An application was conducted by LabVIEW simulation program using Runge-Kutta 2 method. Correlations were analyzed on all parameters of motion equations; four slide mode controllers were implemented to stabilize the altitude and attitude. The stability and disturbances free as possible in our response with multiple set points taken into account. The tuning process achieved after multi attempts of experiments. Future work is to develop a precise trajectory control by using robust techniques to stabilize the whole system and drive the Minicopter to the desired trajectory of Cartesian position, attitude, and airspeed.

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